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# ON THE UNIQUENESS OF REPRESENTATIONAL INDICES OF DERIVATIONS OF $C^*$-ALGEBRAS

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**The paper considers some sufficient conditions for a closed $^*$-derivation of a $C^*$-algebra, implemented by a symmetric operator, to have a unique representational index.**

**1. Introduction.** Let $\mathscr{A}$ be a $C^*$-subalgebra of the algebra $B(H)$ of all bounded operators on a Hilbert space $H$, and let a dense $^*$-subalgebra $D(\delta)$ of $\mathscr{A}$ be the domain of a closed $^*$-derivation $\delta$ from $\mathscr{A}$ into $B(H)$. A closed operator $S$ on $H$ *implements* $\delta$ if $D(S)$ is dense in $H$ and if

$$AD(S) \subseteq D(S) \quad \text{and}$$
$$\delta(S)|_{D(S)} = i(SA - AS)|_{D(S)} \quad \text{for all } A \in D(\delta).$$

If $S$ is symmetric (dissipative), it is called a *symmetric* (*dissipative*) *implementation* of $\delta$. If a closed operator $T$ extends $S$ and also implements $\delta$, then $T$ is called a $\delta$-extension of $S$. If $S$ has no $\delta$-extension, it is called a *maximal implementation* of $\delta$.

If $\delta$ is implemented by a closed operator, it always has an infinite set $\mathscr{I}(\delta)$ of implementations. However, not much can be said about the structure of $\mathscr{I}(\delta)$. We do not even know whether it has maximal implementations. The subsets $\mathscr{S}(\delta)$ and $\mathscr{D}(\delta)$ of $\mathscr{I}(\delta)$ ($\mathscr{S}(\delta) \subseteq \mathscr{D}(\delta)$), which consist respectively of symmetric and of dissipative implementations of $\delta$, are more interesting. In **[4]** it was shown that every symmetric implementation of $\delta$ extends to a *maximal* symmetric implementation of $\delta$. Therefore if $\mathscr{S}(\delta) \neq \varnothing$, then $\mathscr{S}(\delta)$ as well as the set $\mathscr{MS}(\delta)$ of all maximal symmetric implementations of $\delta$ are infinite sets.

If $S \in \mathscr{MS}(\delta)$ and it is not selfadjoint, then the question arises as to whether $S$ has *dissipative* $\delta$-extensions and, if so, whether there exist maximal dissipative implementations of $\delta$. This question was partly answered in **[5]** where it was established that, under some conditions on $\delta$ and $S$ (for example, if $\max(n_-(S), n_+(S)) < \infty$), the maximal dissipative implementations of $\delta$ do exist.