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SPIN MODELS FOR LINK POLYNOMIALS, STRONGLY REGULAR GRAPHS AND JAEGER'S HIGMAN-SIMS MODEL

PIERRE DE LA HARPE

We recall first some known facts on Jones and Kauffman polynomials for links, and on state models for link invariants. We give next an exposition of a recent spin model due to F. Jaeger and which involves the Higman-Sims graph. The associated invariant assigns to an oriented link the evaluation for $a = -\tau^5$ and $z = 1$ of its Kauffman polynomial in the Dubrovnik form, where τ denotes the golden ratio.

1. Introduction. A *knot* is a simple closed curve in $\mathbb{R}^3$ and a *link* is a finite union of disjoint knots. We denote by $\vec{L}$ a link L together with an *orientation* on each of its components. Two oriented links $\vec{L}$, $\vec{L}'$ are *isotopic*, and we write $\vec{L}' \approx \vec{L}$, if there exists a family $(\phi_t)_{0\leq t\leq 1}$ of homeomorphisms of $\mathbb{R}^3$ such that the map $[0, 1] \to \mathbb{R}^3$ sending t to $\phi_t(x)$ is continuous for each $x \in \mathbb{R}^3$ and such that $\phi_0 = \mathrm{id}$, $\phi_1(\vec{L}) = \vec{L}'$, where the last equation indicates that orientations correspond via ϕ. Links considered here are always assumed to be *tame*, namely isotopic to links made of smoothly embedded curves. A Ω-*valued invariant* for oriented links is a map $\vec{L} \mapsto I(\vec{L})$ which associates to each oriented link $\vec{L}$ in $\mathbb{R}^3$ an element $I(\vec{L})$ of some ring Ω, for example $\mathbb{C}$ or a ring of Laurent polynomials, in such a way that $I(\vec{L}') = I(\vec{L})$ whenever $\vec{L}' \approx \vec{L}$.

Classically, one of the most studied example of link invariant is the *Alexander-Conway polynomial* $\Delta(L) \in \mathbb{Z}[t^{\pm 1}]$ defined by J. W. Alexander in 1928 [**Ale**], with a normalization made precise by J. H. Conway in 1969 [**Con**]; the notation (L rather than $\vec{L}$) indicates that, at least for knots, $\Delta(L)$ does not depend on the choice of an orientation on the knot. The polynomial invariant $L \to \Delta(L)$ is well understood in terms of *standard algebraic topology* (homology of "the" infinite cyclic covering of the complement of L in $\mathbb{R}^3$); see e.g. [**Rha**], [**Rol**] or [**BuZ**].

The subject entered a new era in 1984 [**Jo1**] with the discovery of the *Jones polynomial* $V(\vec{L}) \in \mathbb{Z}[t^{\pm 1/2}]$. This was the starting point of several other invariants, including the *Kauffman polynomial*