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BRAIDED GROUPS OF HOPF ALGEBRAS OBTAINED BY TWISTING

D. GUREVICH AND SHAHN MAJID

It is known that every quasitriangular Hopf algebra H can be converted by a process of transmutation into a braided group $B(H, H)$. The latter is a certain braided-cocommutative Hopf algebra in the braided monoidal category of H-modules. We use this transmutation construction to relate two approaches to the quantization of enveloping algebras.

Specifically, we compute $B(\widetilde{H}, \widetilde{H})$ in the case when $\widetilde{H}$ is the quasitriangular Hopf algebra (quantum group) obtained by Drinfeld's twisting construction on a cocommutative Hopf algebra H. In the case when $\widetilde{H}$ is triangular we recover the S-Hopf algebra H_F previously obtained as a deformation-quantization of H. Here H_F is a Hopf algebra in a symmetric monoidal category. We thereby extend the definition of H_F to the braided case where $\widetilde{H}$ is strictly quasitriangular. We also compute its structure to lowest order in a quantization parameter $\hbar$. In this way we show that $B(U_q(g), U_q(g))$ is the quantization of a certain generalized Poisson bracket associated to the Drinfeld-Jimbo solution of the classical Yang-Baxter equations.

1. Introduction. Hopf algebras in braided monoidal categories have been introduced in [10] and [11] in the context of Tannaka-Krein reconstruction theorems. It is shown there that every quantum group gives rise to a Hopf algebra in a braided category by a process of transmutation. The category is that of representations of the quantum group, and in this braided category the resulting Hopf algebra is in a certain sense "braided-cocommutative", i.e. like a group algebra. Hence such Hopf algebras in braided categories are called braided groups. The process is called transmutation because it turns a quantum group in the ordinary category of Hopf algebras into a group-like object in a non-commutative category.

Hopf algebras in symmetric monoidal categories, on the other hand, arise naturally in the deformation-quantization of triangular solutions of the classical Yang-Baxter equations (CYBE) [6]. They have been called S-Hopf algebras and are the enveloping algebras of S-Lie algebras and S-groups. Hence we are led to consider if braided groups, too, can arise as such deformation-quantization of some kind of Poisson structure. This is one motivation for the present paper. The