PACIFIC JOURNAL OF MATHEMATICS
Vol. 162, No. 1, 1994

A CLASSIFICATION OF CERTAIN 3-DIMENSIONAL CONFORMALLY FLAT EUCLIDEAN HYPERSURFACES

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This paper deals with conformally flat hypersurfaces of the 4-dimensional Euclidean space E^4. We classify those conformally flat hypersurfaces of E^4 whose mean curvature vector, H, is an eigenvector of their Laplacian i.e. $\Delta H = \lambda H$; $\lambda \in R$.

The classification is done by proving that the classical Cartan-Schouten result remains valid for this kind of hypersurfaces.

1. Introduction. A Riemannian manifold (M^n, g) is conformally flat, if every point has a neighborhood which is conformal to an open set in the Euclidean space. A submanifold of the Euclidean space E^{n+1} is said to be conformally flat if so it is with respect to the induced Riemannian structure. Thus, in the highest codimension, we can talk about conformally flat hypersurfaces M^n of the Euclidean space E^{n+1}. We classify conformally flat Euclidean hypersurfaces in E^4 whose mean curvature vector is an eigenvector of their Laplacian. More concretely we prove that if $x : M \to E^4$ is a complete conformally flat hypersurface immersed in the 4-dimensional Euclidean space, which satisfies $\Delta H = \lambda H$, H being the mean curvature vector of the immersion, then it is either minimal or the Riemannian product $E^p \times S^{(3-p)}$, $0 \leq p \leq 3$.

The dimension of the hypersurface seems to play an important role in the study of conformally flat Euclidean hypersurfaces. For $n = 2$, the existence of isothermal coordinates means that any Riemannian surface is conformally flat. We shall discuss the case $n = 3$ at the end of this section. For $n \geq 4$, the following result of Cartan-Schouten, [2], [27], is of fundamental importance: If M^n is a hypersurface immersed in E^{n+1}, $n \geq 4$, then M^n is conformally flat in the induced metric, if and only if, at least $n-1$ of the principal curvatures coincide at each point. (See also Nishikawa and Maeda [24].) Using this theorem Kulkarni [20] and Nishikawa and Maeda [24] gave a local classification of conformally flat hypersurfaces in E^{n+1}, $n \geq 4$.

Unfortunately canal hypersurfaces [4, p. 166] are conformally flat hypersurfaces which do not fall under this classification. Also, Cecil