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SEMISIMPLICITY OF RESTRICTED ENVELOPING ALGEBRAS OF LIE SUPERALGEBRAS

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Let $L = L_0 \oplus L_1$ be a restricted Lie superalgebra over a field of characteristic $p > 2$. We let $u(L)$ denote the restricted enveloping algebra of L and we will be concerned with when $u(L)$ is semisimple, semiprime, or prime.

The structure of $u(L)$ is sufficiently close to that of a Hopf algebra that we will obtain ring theoretic information about $u(L)$ by first applying basic facts about finite dimensional Hopf algebras to Hopf algebras of the form $u(L)\# G$. Our main result along these lines is that if $u(L)$ is semisimple with L finite dimensional, then $L_1 = 0$. Combining this with a result of Hochschild, we will obtain a complete description of those finite dimensional L such that $u(L)$ is semisimple.

In the infinite dimensional case, we will obtain various necessary conditions for $u(L)$ to be prime or semiprime.

Introduction. Let $L = L_0 \oplus L_1$ be a restricted Lie superalgebra over a field K of characteristic $p > 2$. We let $u(L)$ denote the restricted enveloping algebra of L and we will be concerned with when $u(L)$ is semisimple, semiprime, or prime.

When $L_1 \neq 0$, $u(L)$ is not a Hopf algebra. However the structure of $u(L)$ is sufficiently close to that of a Hopf algebra that we can construct a skew group ring $u(L)\# G$ which is a Hopf algebra. We will obtain ring theoretic information about $u(L)$ by first applying basic facts about finite dimensional Hopf algebras to $u(L)\# G$. Our main result along these lines is

THEOREM. *If L is finite dimensional such that $u(L)$ is semisimple then $L_1 = 0$.*

Combining this theorem with Hochschild's theorem [H] on the semisimplicity of $u(L_0)$, it easily follows that

COROLLARY. *If L is finite dimensional then $u(L)$ is semisimple if and only if $L_1 = 0$, L is abelian, and the pth power map on L_0 is injective.*