PACIFIC JOURNAL OF MATHEMATICS
Vol. 162, No. 2, 1994

DEC GROUPS FOR ARBITRARILY HIGH EXPONENTS

B. A. SETHURAMAN

For each prime p and each $n \geq 1$ ($n \geq 2$ if $p = 2$), examples are constructed of a Galois extension K/F whose Galois group has exponent p^n and a central simple F-algebra A of exponent p which is split by K but is not in the Dec group of K/F.

1. Introduction. Let K/F be an abelian Galois extension of fields, and let $G = \mathscr{G}(K/F)$. Let $G = G_1 \times G_2 \times \cdots \times G_k$ be a direct sum decomposition of G into cyclic groups, with $G_i = \langle \sigma_i \rangle$ ($i = 1, \ldots, k$). Let F_i be the fixed field of $G_1 \times \cdots \times G_{i-1} \times G_{i+1} \times \cdots \times G_k$ ($i = 1, \ldots, k$). Thus, the F_i are cyclic Galois extensions of F, with Galois group isomorphic to G_i. The group $\mathrm{Dec}(K/F)$ is defined as the subgroup of $\mathrm{Br}(K/F)$ generated by the subgroups $\mathrm{Br}(F_i/F)$ ($i = 1, \ldots, k$). This group was introduced by Tignol ([**T1**]), where he shows that $\mathrm{Dec}(K/F)$ is independent of the choice of the direct sum decomposition of G. If p is a prime, we will write ${}_{p^n}\mathrm{Br}(K/F)$ and ${}_{p^n}\mathrm{Dec}(K/F)$ for the subgroups of $\mathrm{Br}(K/F)$ and $\mathrm{Dec}(K/F)$ consisting of all elements with exponent dividing p^n.

A key issue in several past constructions of division algebras has been the non-triviality of the factor group ${}_p\mathrm{Br}(K/F)/{}_p\mathrm{Dec}(K/F)$ for suitable abelian extensions K/F. For instance, the Amitsur-Rowen-Tignol construction of an algebra of index 8 with involution with no quaternion subalgebra ([**ART**]) depends crucially on the existence of a triquadratic extension K/F for which ${}_2\mathrm{Br}(K/F) \neq {}_2\mathrm{Dec}(K/F)$. Similarly, the constructions of indecomposable algebras of exponent p by Tignol ([**T2**]) and Jacob ([**J**]) also depend on the existence of an (elementary) abelian extension K/F for which ${}_p\mathrm{Br}(K/F) \neq {}_p\mathrm{Dec}(K/F)$.

The extension fields K/F that occur in these examples above are all of exponent p, and it is an interesting question whether there exist abelian extensions K/F whose Galois groups have arbitrarily high (p-power) exponents for which the factor group ${}_p\mathrm{Br}(K/F)/{}_p\mathrm{Dec}(K/F)$ is non-trivial. The purpose of this paper is to show that for each $n \geq 1$ ($n \geq 2$ if $p = 2$), there exists an abelian extension K/F with Galois group $\mathbb{Z}/p^n\mathbb{Z} \times \mathbb{Z}/p\mathbb{Z}$ (and thus, of exponent p^n) and an algebra $A \in {}_p\mathrm{Br}(K/F)$ such that $A \notin {}_p\mathrm{Dec}(K/F)$. (Note that if K/F is an