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WHEN L^1 OF A VECTOR MEASURE IS AN AL-SPACE

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We consider the space of real functions which are integrable with respect to a countably additive vector measure with values in a Banach space. In a previous paper we showed that this space can be any order continuous Banach lattice with weak order unit. We study a priori conditions on the vector measure in order to guarantee that the resulting L^1 is order isomorphic to an AL-space. We prove that for separable measures with no atoms there exists a c_0-valued measure that generates the same space of integrable functions.

Introduction. Given a vector measure ν we consider the space of classes of real functions which are integrable with respect to ν in the sense of Lewis **[L-1]**, denoted by $L^1(\nu)$. In **[C**, Theorem 8] we showed that every order continuous Banach lattice with weak unit can be obtained as $L^1(\nu)$ for a suitable vector measure ν. In particular we have Hilbert spaces as L^1 of a vector measure. A natural question arises. Under what conditions on the vector measure, or on the Banach space in which the measure takes its values, is the resulting L^1 of the vector measure order isomorphic to an AL-space? Recall that a Banach lattice is an *AL-space* when the norm is additive for disjoint vectors. An *order isomorphism* is a linear isomorphism that preserves the lattice operations. So the question can be restated in the following way. When can $L^1(\nu)$ be equivalently renormed so that endowed with the new norm and the same order is a Banach lattice where the norm is additive for disjoint functions?

In §1 we fix notation and basic definitions.

In §2 we show the special role that the space $L^1(|\nu|)$ plays in the problem we are studying, where $|\nu|$ is the variation of ν. It is shown in Proposition 2 that $L^1(\nu)$ is an $\mathscr{L}_1$-space, in the sense of Lindenstrauss and Pelczynski **[L-P]**, if and only if it is order isomorphic to $L^1(|\nu|)$. From this it follows that bounded variation of the measure is a necessary condition. The conditions cannot be placed on the Banach space in which the measure takes its values. This is shown in Example 1, that also shows that neither bounded variation nor domination of the variation by the semivariation are sufficient conditions.

In §3 we study measures with values in $C(K)$ spaces. In Theorem 1