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COMMUTANTS OF TOEPLITZ OPERATORS ON THE BERGMAN SPACE

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This paper describes the commutants of certain analytic Toeplitz operators. To underline the difference between the Bergman and Hardy spaces, we first prove that on the Bergman space L_a^2 the only isometric Toeplitz operators with harmonic symbols are scalar multiples of the identity. If T denotes the norm closed subalgebra of $\mathrm{L}(L_a^2)$ generated by Toeplitz operators, we show that for each positive integer n, $\{T_{z^n}\}' \cap \mathrm{T}$ is the set of all analytic Toeplitz operators. This result is also valid for the Hardy space. Here $\{T_{z^n}\}'$ denotes the commutant of T_{z^n}. Finally we prove the analogous result for T_{u^n}, where u is an analytic, one-to-one map of the unit disk onto itself.

Introduction. Let D denote the open unit disk in the complex plane and let dA denote the usual Lebesgue area measure on D. The complex space $L^2(D, dA)$ is a Hilbert space with the inner product

$$\langle f, g\rangle = \int_D f\bar{g}\, dA.$$

The Bergman space L_a^2 is the set of those functions in $L^2(D, dA)$ that are analytic on D. The Bergman space is a closed subspace of $L^2(D, dA)$, and so there is an orthogonal projection P from $L^2(D, dA)$ onto L_a^2. For $\varphi \in L^\infty(D, dA)$, the Toeplitz operator with symbol φ, denoted T_φ, is the operator from L_a^2 to L_a^2 defined by $T_\varphi f = P(\varphi f)$. For more information about the Bergman space and its operators see [**4**].

The algebra of bounded analytic functions on D will be denoted by H^∞. If $\varphi \in H^\infty$, then T_φ is called an analytic Toeplitz operator.

For a Hilbert space H, $\mathrm{L}(H)$ denotes the algebra of all bounded linear operators on H. If $S \subset \mathrm{L}(H)$, then $S' = \{B \in \mathrm{L}(H) : AB = BA \text{ for all } A \in S\}$ is the commutant of S. In this paper we are interested in finding commutants of certain analytic Toeplitz operators acting on the Bergman space.

Much work has been done in studying commutants of analytic Toeplitz operators on the Hardy space. Some of those results can be extended to the Bergman space case. The complex space $L^2(\partial D)$