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ON THE EXISTENCE OF CONVEX CLASSICAL SOLUTIONS TO MULTILAYER FLUID PROBLEMS IN ARBITRARY SPACE DIMENSIONS

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We study certain multilayer free-boundary problems, in which the layer interfaces constitute a nested family of convex closed surfaces, each characterized by a Bernoulli joining condition between the potentials in the neighboring layers. In this context, we develop convex variational methods based on a family of convexity-preserving free-boundary perturbation operators, and we apply these methods in the study of the existence of convex solutions.

1. Introduction. The main purpose of this paper is to apply convex variational techniques to study the question of the existence of convex classical solutions to certain multiple-free-boundary problems arising in fluid dynamics, called multilayer fluid problems.

1.1. *Problem.* In $\mathbb{R}^m$, $m \geq 2$, let an annular domain Ω of the form $\Omega = D^+\backslash\text{Cl}(D^-)$ be given, where $D^\pm$ are fixed, bounded, simply-connected, nested C^1-domains. Given $n \in \mathbb{N}$ and continuous functions $\lambda_i(x)\colon \text{Cl}(D^+) \to \mathbb{R}$, $i = 1, 2, \ldots, n$, we seek a nested family of C^1-domains $D_1, D_2, \ldots, D_n$ (with boundaries $\Gamma_i = \partial D_i$) such that $\text{Cl}(D_i) \subset D_{i+1}$ for $i = 0, \ldots, n$, (where we set $D_0 = D^-$ and $D_{n+1} = D^+$) and such that

$$|\nabla U_i|^2 = |\nabla U_{i+1}|^2 + \lambda_i(x) \quad \text{on } \Gamma_i \tag{1.1}$$

for $i = 1, \ldots, n$, where $U(x)$ solves the boundary value problem

$$\Delta U = 0 \quad \text{in } \Omega\backslash(\Gamma_1 \cup \cdots \cup \Gamma_n), \qquad U(\Gamma_i) = i \quad \text{for } i = 0, 1, \ldots, n+1, \tag{1.2}$$

and where, for each i, U_i denotes the restriction of U to the closure of the annular domain $\Omega_i := D_i\backslash\text{Cl}(D_{i-1})$ with boundary $\partial\Omega_i = \Gamma_i \cup \Gamma_{i-1}$.

1.2. *Problem.* This denotes the modified version of Problem 1.1 in which Γ_0 becomes a free boundary characterized by the requirement that

$$|\nabla U_1| = a_0(x) \quad \text{on } \Gamma_0. \tag{1.3}$$