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THE CLASSIFICATION OF COMPLETE LOCALLY CONFORMALLY FLAT MANIFOLDS OF NONNEGATIVE RICCI CURVATURE

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The main purpose of this note is to give a classification of complete locally conformally flat manifolds of nonnegative Ricci curvature. Such classification for the compact case has been obtained by various authors in the past decade.

1. Introduction. Recall that an n-dimensional Riemannian manifold (M^n, g) is said to be locally conformally flat if it admits a coordinate covering $\{U_\alpha, \phi_\alpha\}$ such that the map $\phi_\alpha: (U_\alpha, g_\alpha) \to (S^n, g_0)$ is a conformal map, where g_0 is the standard metric on S^n. It follows from this definition that the Weyl tensor of g vanishes. In particular, the full curvature tensor of g can be recovered from the Ricci tensor of g (an alternating sum). Thus conditions on the Ricci tensor of such manifolds impose very strong restrictions on their metrics. In the first part of this note we confirm this by showing,

THEOREM 1. *If (M^n, g) is a complete locally conformally flat Riemannian manifold with* $\mathrm{Ric}(g) \geq 0$, *then the universal cover $\widetilde{M}$ of M with the pulled-back metric is either conformally equivalent to S^n, R^n or is isometric to $R \times S^{n-1}$. If M itself is compact, then $\widetilde{M}$ is either conformally equivalent to S^n or isometric to R^n, $R \times S^{n-1}$, where S^n and S^{n-1} are spheres of constant curvature.*

The second part of Theorem 1 was obtained by various authors as consequences of investigating more general classes of manifolds, see the work of Schoen and Yau ([**SY**]) for references. An elementary proof for this case was also given recently by Noronha ([**No**]).

We remark that although the validity of Theorem 1 is not surprising, many similar problems in Riemannian geometry still remain open in the noncompact case, while the compact case has long been solved. The difficulty usually lies in the lack of analytic techniques for noncompact manifolds. The analysis in our case does carry through ([**SY**]) essentially because of the developing map as outlined below.