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CURVATURE CHARACTERIZATION OF CERTAIN BOUNDED DOMAINS OF HOLOMORPHY

FANGYANG ZHENG

In this note, we study the relation between the existence of a negatively curved complete hermitian metric on a complex manifold M and the product structure of (or contained in) M. We introduce the concept of geometric ranks and give a curvature characterization of the rank one manifolds, which generalizes the previous results of P. Yang and N. Mok (see below). In the proof, we used the old techniques of Yau's Schwartz lemma and Cheng-Yau's result on the existence of Kähler-Einstein metrics.

1. Introduction and statement of results. Let $M = M_1 \times M_2$ be the product of two complex manifolds. Then it is generally believed that M does not admit any complete Kähler metric with bisectional curvature bounded between two negative constants. When M is compact, this is certainly true since the cotangent bundle T^*_M is not ample. In the noncompact case, the first result toward this direction was obtained by Paul Yang in 1976:

THEOREM ([Y]). *For any $n \geq 2$, there exists no complete Kähler metric on the polydisc C^n with bisectional curvature bounded between two negative constants.*

In [M], as an application of his metric rigidity theory, Mok generalized the above to give an interesting curvature characterization of the rank one bounded symmetric domains:

THEOREM ([M]). *If Ω is a bounded symmetric domain of rank ≥ 2, then there exists no complete hermitian metric on Ω with bounded torsion and with bisectional curvature bounded between two negative constants.*

Mok's proof is a constructive one. It used the existence of a uniform lattice Γ on Ω, as well as the integral formula on Ω/Γ discovered by Mok (cf. Proposition (3.2) in [M]). This proof is very interesting by itself. However, we noticed that Yang's approach can be used to give a more straightforward proof of Mok's result, and the conclusion holds