PACIFIC JOURNAL OF MATHEMATICS
Vol. 163, No. 1, 1994

INTERPOLATED FREE GROUP FACTORS

KEN DYKEMA

The interpolated free group factors $L(\mathbf{F}_r)$ for $1 < r \leq \infty$ (also defined by F. Rădulescu) are given another (but equivalent) definition as well as proofs of their properties with respect to compression by projections and free products. In order to prove the addition formula for free products, algebraic techniques are developed which allow us to show $R * R \cong L(\mathbf{F}_2)$ where R is the hyperfinite II_1-factor.

Introduction. The free group factors $L(\mathbf{F}_n)$ for $n = 2, 3, \ldots, \infty$ (introduced in [**4**]) have recently been extensively studied [**11**, **2**, **5**, **6**, **7**] using Voiculescu's theory of freeness in noncommutative probability spaces (see [**8**, **9**, **10**, **11**, **12**, **13**], especially the latter for an overview). One hopes to eventually be able to solve the isomorphism question, first raised by R. V. Kadison of whether $L(\mathbf{F}_n) \cong L(\mathbf{F}_m)$ for $n \neq m$. In [**7**], F. Rădulescu introduced II_1-factors $L(\mathbf{F}_r)$ for $1 < r \leq \infty$, equalling the free group factor $L(\mathbf{F}_n)$ when $r = n \in \mathbf{N}\backslash\{0, 1\}$ and satisfying

$$(1) \qquad L(\mathbf{F}_r) * L(\mathbf{F}_{r'}) = L(\mathbf{F}_{r+r'}) \qquad (1 < r, r' \leq \infty)$$

and

$$(2) \qquad L(\mathbf{F}_r)_\gamma = L\left(\mathbf{F}\left(1 + \frac{r-1}{\gamma^2}\right)\right) \qquad (1 < r \leq \infty,\ 0 < \gamma < \infty),$$

where for a II_1-factor $\mathscr{M}$, $\mathscr{M}_\gamma$ means the algebra [**4**] defined as follows: for $0 < \gamma \leq 1$, $\mathscr{M}_\gamma = p\mathscr{M}p$, where $p \in \mathscr{M}$ is a selfadjoint projection of trace γ; for $\gamma = n = 2, 3, \ldots$ one has $\mathscr{M}_\gamma = \mathscr{M} \otimes M_n(\mathbf{C})$; for $0 < \gamma_1, \gamma_2 < \infty$ one has

$$\mathscr{M}_{\gamma_1\gamma_2} = (\mathscr{M}_{\gamma_1})_{\gamma_2}.$$

We had independently found the interpolated free group factors $L(\mathbf{F}_r)$ $(1 < r \leq \infty)$ and the formulas (1) and (2), defining them differently and using different techniques. In this paper we give our definition and proofs. This picture of $L(\mathbf{F}_r)$ is sometimes more convenient, e.g. §4 of [**3**]. It is a natural extension of the result [**2**] that

$$(3) \qquad \mathrm{L}(\mathbf{Z}) * R \cong \mathrm{L}(\mathbf{F}_2),$$