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ON AMBIENTAL BORDISM

CARLOS BIASI

Let M^m be a closed and oriented submanifold of a closed or oriented manifold N^n, such that $[M, i] = 0 \in \Omega_m(N)$, where $i: M \to N$ is the inclusion and $\Omega_m(N)$ is the mth oriented bordism group of N. If $n = m + 2$ or $m \leq 3$ or $m \leq 4$ and $n \neq 7$ then M bounds in N.

Introduction. Let us consider M^m a closed submanifold of N^n. In this paper, we study the possibility that there exists submanifold $W^{m+1} \subset N^n$ such that $\partial W = M$. If $M = S^m$ and $N = S^{m+2}$, such that a submanifold W is called a Seifert surface knot S^m. In [**5**], Sato showed that every connected closed and oriented submanifold M^m of S^{m+2} is a boundary of an oriented surface of S^{m+2}.

In [**4**], Hirsch studies the following problem: If a compact connected and oriented manifold M^m bounds, does there exist embedding from M^m into $\mathbb{R}^n$ which is a boundary in $\mathbb{R}^n$?

The answer is yes, if $n \geq 2m$.

The difference between the two problems is that, in our case, the embedding from M into N is fixed.

There is an obvious necessary condition for the existence of W, when M and N are oriented manifolds.

Let $\Omega_m(N)$ be the mth oriented bordism group of N [**2**]. If $i: M \to N$ is the inclusion map, we can define an element $[M, i]$ in $\Omega_m(N)$ and see that $[M, i] = 0$ if M bounds in N.

Generally, the converse in not true, but sometimes the vanishing of $[M, i]$ guarantees the existence of W, for example if the codimension $n - m$ is large.

We prove the following theorem.

THEOREM 5.2. *Let us suppose that $M^m \subset N^n$, $n > m + 1$, is such that $[M, i] = 0$ in $\Omega_m(N)$. Then M bounds in N if one of the following conditions occurs*:

(a) $n = m + 2$,
(b) $m \leq 3$,
(c) $m \leq 4$ *and* $n \neq 7$.