A TRANSVERSE STRUCTURE
FOR THE LIE-POISSON BRACKET
ON THE DUAL OF THE VIRASORO ALGEBRA

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KdV equations can be described as Hamiltonian systems on the dual of the Virasoro algebra with the canonical Lie-Poisson (also called Berezin-Kirillov-Kostant) bracket. In this paper we give an explicit transverse structure for this Poisson manifold along a finite dimensional submanifold. The structure is linearizable and equivalent to the Lie-Poisson structure on $\mathrm{sl}(2, \mathbf{R})^*$. This problem is closely related to the classification of Hill's equations.

1. Introduction and main definitions. It was known since Lie's time that if a manifold has a Poisson structure and the rank of the Poisson tensor is constant around a point (that is, the point is *regular*), then the manifold can be locally described at such a point as foliated into leaves of maximum rank or *symplectic* leaves. If the Poisson manifold is the dual of a Lie algebra with its *Lie-Poisson bracket*, then the symplectic leaves coincide with the orbits under the coadjoint action of the group. If the point is singular the local description can be achieved by finding a section which is transversal to the orbit of the point and which is endowed with a Poisson structure induced by the global Poisson bracket. This induced bracket, or *transverse structure*, was initially introduced by A. Weinstein for finite dimensional Poisson manifolds (see [20]) and it describes the relation between the symplectic structures on the different leaves as we cross them transversally to the orbit of a singular point. Weinstein also proved that transverse structures were unique in the following sense: if we have two sections transversal to the orbit of a singular point with Poisson brackets induced on them and with dimensions equal to the codimension of the orbit, then there exists a Poisson isomorphism of the manifold, defined between two neighbourhoods of the intersections with the orbit, which will clearly preserve the two transverse structures.

The aim of this paper is to show the geometrical description of the coadjoint orbits on the dual of the Virasoro algebra as we move transversally through them and to use this description to find an explicit transverse structure for its Lie-Poisson bracket. Descriptions