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FANO BUNDLES AND SPLITTING THEOREMS ON PROJECTIVE SPACES AND QUADRICS

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Introduction. In this paper rank 2 vector bundles E on projective spaces $\mathbb{P}_n$ and quadrics Q_n are investigated which enjoy the additional property that their projectized bundles $\mathbb{P}(E)$ are Fano manifolds, i.e. have negative canonical bundles. Such bundles are shortly called Fano bundles. Up to dimension 3 Fano bundles are completely classified by **[SW]**, **[SW′]**, **[SW″]**, **[SSW]**. The aim of this paper is to describe the structure of Fano bundles in dimension ≥ 4. Namely we prove the following

MAIN THEOREM. *Let E be a rank 2 Fano bundle on $\mathbb{P}_n$ or Q_n, $n \geq 4$. Then up to some explicit exceptions on Q_4 and Q_5 (see ex. (2.1), (2.2), (2.3)), E splits into a direct sum of line bundles.*

A rank 2 bundle E on $\mathbb{P}_n$ is Fano if and only if the "$\mathbb{Q}$-vector bundle" $E \otimes (\det E^*)/2 \otimes \mathscr{O}(\frac{n+1}{2})$ is ample, i.e.

$$\mathscr{O}_{\mathbb{P}(E)}(2) \otimes \pi^* \left(\det E^* \otimes \mathscr{O}\left(\frac{n+1}{2}\right)\right) \quad \text{is ample}.$$

If we normalize E in the following sense: $E_0 = E \otimes (\det E^*)/2$, so that $c_1(E_0) = 0$; then E is Fano iff $E_0(\frac{n+1}{2})$ is ample. Similarly on quadrics. In other words, we show that bundles with $E_0(\frac{n+1}{2})$ ample must split (on $\mathbb{P}_n$, $n \geq 4$). In other words: ample bundles with $c_1(E) \leq n + 1$ split.

We prove even more:

THEOREM (9.1). *Let F be an ample 2-bundle on $\mathbb{P}_n$. Then F splits if one of the following assumptions hold:*

(1) $n = 4$, $c_1(F) \leq 6$,
(2) $n = 5$, $c_1(F) \leq 8$,