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# WEIGHTED MAXIMAL FUNCTIONS AND DERIVATIVES OF INVARIANT POISSON INTEGRALS OF POTENTIALS

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**In this paper we prove $L^p$ estimates for weighted maximal functions of invariant Poisson integrals of potentials. From this it follows that the exceptional sets of the Poisson integrals of potentials are sets of zero Hausdorff capacity.**

Let $S$ denote the boundary of $B_n$, the unit ball in $C^n$, and let $d\sigma$ be the unusual rotation invariant measure defined on $S$. If $g$ is a function belonging to the usual Lebesgue space $L^1(d\sigma)$ of functions defined on the sphere then by $P[g]$ we will mean the invariant Poisson integral of $g$ defined by the equation

$$P[g](z) = \int_S g(\eta)\frac{(1-|z|^2)^n}{|1-\langle z, \eta\rangle|^{2n}}\, d\sigma(\eta),$$

where $z \in B_n$.

In this paper we will continue the work of Ahern and Cascante [**ACa**] and study invariant Poisson integrals of potentials of distributions in the atomic Hardy spaces $H^p_{at}$ where $0 < p \leq 1$. Precisely, if $v$ denotes a distribution in the space $H^p_{at}$ defined by Garnett and Latter and if $0 < \beta < n$ and $\zeta \in S$ define the (non-isotopic) potential of $v$ by

$$I_\beta v(\zeta) = \int_S v(\eta)\frac{d\sigma(\eta)}{|1-\langle \zeta, \eta\rangle|^{n-\beta}}.$$

Let $f(z) = P[I_\beta v](z)$ and denote by $f^*_\alpha$ the admissible maximal function of $f$ defined on the sphere $S$ associated with the admissible approach region of aperture $\alpha$. Thus, for each fixed $\alpha > 1$

$$f^*_\alpha(\zeta) = \sup_{w\in\Gamma_\alpha(\zeta)} |f(w)|,$$

where $\Gamma_a(\zeta)$ is the admissible approach region

$$\Gamma_\alpha(\zeta) = \{w \in B_n : |1-\langle w, \zeta\rangle| < \tfrac{\alpha}{2}(1-|w|^2)\}.$$

Suppose that $\mu$ is a positive measure on $S$ satisfying the condition

$$(1) \qquad \mu(B(\zeta; \delta)) \leq C\delta^{n-\beta p}$$