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THE COHOMOLOGY RING OF THE SPACES OF LOOPS ON LIE GROUPS AND HOMOGENEOUS SPACES

KATSUHIKO KURIBAYASHI

Let X be a simply connected space whose $\bmod p$ cohomology is isomorphic to that of a compact, simply connected, simple Lie group as an algebra over the Steenrod algebra. We determine the algebra structure of the $\bmod p$ cohomology of ΩX algebraically. Moreover we give a method to determine the algebra structure of the $\bmod p$ cohomology of the space of loops on a homogeneous space.

0. Introduction. Let G be a compact simply connected Lie group and ΩX the space of loops on a space X. In [**4**], R. Bott has given a method to obtain generators of the Pontryagin ring $H_*(\Omega G)$ and has determined its Hopf algebra structure explicitly for $G = \mathrm{SU}(m)$, $\mathrm{Spin}(m)$ and G_2. By applying this method, T. Watanabe [**23**] has determined the Hopf algebra structure of $H_*(\Omega F_4)$. A. Kono and K. Kozima [**8**] have determined the Hopf algebra structure over the Steenrod algebra $\mathscr{A}(2)$ of $H_*(\Omega G; \mathbb{Z}/2)$ for $G = F_4, E_6, E_7$ and E_8, without using Bott's method. In order to determine the algebra structure, they have made use of the Eilenberg-Moore spectral sequence [**16**] which converges to $H^*(G; \mathbb{Z}/2)$ and whose E_2-term is isomorphic to $\mathrm{Ext}^{**}_{H_*(\Omega G;\mathbb{Z}/2)}(\mathbb{Z}/2, \mathbb{Z}/2)$. Moreover a homotopy fiber of $\Omega x_4\colon \Omega BG \to \Omega K(\mathbb{Z}, 4)$ has been used to examine the coalgebra structure, where $x_4\colon BG \to K(\mathbb{Z}, 4)$ is a map representing the generator of $H^4(BG)$. The consideration of the dual of those results ([**4**], [**8**], [**23**]) enables us to determine the Hopf algebra structure of the $\bmod p$ cohomology of ΩG for the Lie groups G. On the other hand, we can decide the coalgebra structure of $H^*(\Omega G; \mathbb{Z}/p)$ algebraically from the algebra $H^*(G; \mathbb{Z}/p)$ over the Steenrod algebra $\mathscr{A}(p)$. The following result is due to R. M. Kane [**5**].

THEOREM 0.1. *Suppose that X is a simply connected H-space and* (0.1): *there exists a compact, simply connected, simple Lie group G such that $H^*(X; \mathbb{Z}/p) \cong H^*(G; \mathbb{Z}/p)$ as an algebra over the $\bmod p$ Steenrod algebra $\mathscr{A}(p)$. (We do not require the existence of any map between X and G which induces the isomorphism.)*

Then $H^*(\Omega X; \mathbb{Z}/p) \cong H^*(\Omega G; \mathbb{Z}/p)$ as a coalgebra.