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KNOTS WITH ALGEBRAIC UNKNOTTING NUMBER ONE

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Every knot, K, in S^3 has associated to it an equivalence class of matrices based on S-equivalence of Seifert matrices. When the knot is altered by changing a crossing, the S-equivalence class of the new knot is related to that of the original knot in a very specific way. This change in the Seifert matrices can be studied without regard to the underlying geometric situation, leading to a theory of *algebraic crossing changes*. Thus, the *algebraic unknotting number* may be defined as the smallest number of these algebraic crossing changes necessary to convert a Seifert matrix for the knot into a matrix for the unknot. A straightforward test of some well-known knot invariants will reveal that the algebraic unknotting number is one.

In [**4**], Murakami defined an operation on Seifert matrices that he called an *algebraic unknotting operation.* He showed that any geometric crossing change induced an algebraic unknotting operation on a suitably chosen Seifert matrix. Since any knot could be changed into any other knot by a sequence of crossing changes, any Seifert matrix could be transformed into any other Seifert matrix by a sequence of algebraic unknotting operations and S-equivalences. For knots K_1 and K_2 the *algebraic Gordian distance* from K_1 to K_2 is the minimum number of algebraic unknotting operations needed in such a sequence. The *algebraic unknotting number*, $u_a(K)$, is then the algebraic Gordian distance of K from the unknot, i.e. the minimum number of algebraic unknotting operations needed to reduce a Seifert matrix for K to a matrix S-equivalent to the zero matrix.

Since every crossing change induces an algebraic unknotting operation, there is the inequality $u_a(K) \leq u(K)$ where $u(K)$ is the regular geometric unknotting number of the knot. And in many cases $u_a(K)$ is the appropriate object of study rather than $u(K)$ because only the algebraic information contained in a Seifert matrix is used. Such is the case in Murasugi's result on signatures [**5**] and Nakanishi's theorem about minor indices [**6**]. Also, results depending only on the abelian invariants (notably Lickorish [**3**] as generalized by Cochran and Lickorish [**1**]) apply to $u_a(K)$ since all of the homology information about