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# ASYMPTOTIC RADIAL SYMMETRY FOR SOLUTIONS OF $\Delta u + e^u = 0$ IN A PUNCTURED DISC

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**In this paper a representation formula for solutions of the equation**

$$(*) \qquad \Delta u + 2Ke^u = 0, \quad K \text{ a constant},$$

**in a punctured disc in terms of multi-valued meromorphic functions is found. As application it is deduced that a necessary and sufficient condition for a solution of $(*)$, $K > 0$, being asymptotic radially symmetric is**

$$\int e^u < \infty.$$

**1. Introduction.** In [3], L. A. Caffarelli, B. Gidas, and J. Spruck proved that non-negative smooth solutions of the conformally invariant equation

$$(1) \qquad \Delta u + u^{(n+2)/(n-2)} = 0, \qquad u \geq 0,$$

in a punctured $n$-dimensional ball, $n \geq 3$, with an isolated singularity at the origin, are asymptotically radial. More precisely, if $u$ is a solution of (1), then

$$u(x) = (1 + o(1))\psi(|x|) \quad \text{as } x \to 0,$$

for some radial singular solution $\psi(r)$.

Geometrically speaking, to solve equation (1) is to find locally a conformal metric on a conformally flat $n$-dimensional manifold with constant scalar curvature. Therefore, its two-dimensional analogue is

$$(2) \qquad \Delta u + e^u = 0.$$

In this paper, we shall establish a similar asymptotic radial symmetry result for a smooth solution $u$ of (2) in the punctured disc, $D^* = D\backslash\{0\}$, $D = \{z \in \mathbb{C} \,|\, |z| < 1\}$, with an isolated singularity at the origin, under

$$(3) \qquad \int_{D^*} e^u < +\infty.$$

Unlike the higher dimensional case, as one will see, that the integrability condition (3) is necessary for $u$ being asymptotically radial.