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ON SIEVED ORTHOGONAL POLYNOMIALS X: GENERAL BLOCKS OF RECURRENCE RELATIONS

JAIRO A. CHARRIS, MOURAD E. H. ISMAIL, AND SERGIO MONSALVE

Orthogonal polynomials defined by general blocks of recurrence relations are examined. The connection with polynomial mappings is established, and applications are given to sieved orthogonal polynomials. This work extends earlier work on symmetric sieved polynomials to the case when the polynomials are not necessarily symmetric.

1. Introduction. We study in this paper systems $\{p_n(x)\}$ of orthogonal polynomials defined by general blocks of recurrence relations of the type

$$
\begin{aligned}
(1.1)\quad (x-b_n^{(0)})p_{nk}(x) &= p_{nk+1}(x)+a_n^{(0)}p_{nk-1}(x),\\
&\vdots\\
(x-b_n^{(j)})p_{nk+j}(x) &= p_{nk+j+1}(x)+a_n^{(j)}p_{nk+j-1}(x),\\
&\vdots\\
(x-b_n^{(k-1)})p_{(n+1)k-1}(x) &= p_{(n+1)k}(x)+a_n^{(k-1)}p_{(n+1)k-2}(x),
\end{aligned}
$$

$0\le j\le k-1$, $n\ge 0$, and satisfying initial conditions

$$
(1.2)\qquad p_{-1}(x)=0,\quad p_0(x)=1.
$$

We shall assume $a_n^{(j)}>0$, $j=0,1,\dots,k-1$, $n\ge 0$ and also that $k\ge 2$. Observe that the p_n's do not depend on $a_0^{(0)}$, so we make the convenient choice $a_0^{(0)}=1$. Clearly $\{p_n(x)\}$ is a system of monic orthogonal polynomials.

The case of $b_n^{(j)}=0$, $n\ge 0$, $0\le j\le k-1$, has been treated in a previous paper [9] by Charris and Ismail, where they also assumed that the determinants

$$
(1.3)\ \Delta_n(2,k-1)=\begin{vmatrix}
x & -1 & 0 & 0 & \cdots & 0 & 0\\
-a_n^{(2)} & x & -1 & 0 & \cdots & 0 & 0\\
0 & -a_n^{(3)} & x & -1 & \cdots & 0 & 0\\
\cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot\\
0 & 0 & 0 & 0 & \cdots & a_n^{(k-1)} & x
\end{vmatrix},
\qquad n\ge 0,
$$