PACIFIC JOURNAL OF MATHEMATICS
Vol. 163, No. 2, 1994

TENT SPACES OVER GENERAL APPROACH REGIONS AND POINTWISE ESTIMATES

MARÍA J. CARRO AND JAVIER SORIA

We consider the study of the tent spaces over general (possibly tangential) approach regions and their atomic decomposition. As a consequence, we obtain some pointwise estimates for a class of operators, using the duality properties of a certain type of Carleson measures. In particular, we can get the boundedness of a family of bilinear operators defined on the product of L^q and some space of measures, into a Lipschitz space; we give yet another proof of the pointwise boundedness for the Fourier transform of distributions in H^p and we improve and generalize the Féjer-Riesz inequality for harmonic extensions of H^p functions.

Several authors have studied the boundedness of maximal operators defined by means of general subsets. For example, in **[8]**, a Hardy-Littlewood type operator is associated with a collection of subsets $\Omega_x \subset \mathbf{R}^{n+1}_+$, $x \in \mathbf{R}^n$. The natural way to define the balls for these sets is to take the subset of Ω_x at level t, that is, the set of points $z \in \mathbf{R}^n$ so that $(z, t) \in \Omega_x$. Our idea is to also replace the cone $\Gamma(x) = \{(y, t) \in \mathbf{R}^{n+1}_+ : |x-y| < t\}$ in the definition of the tent spaces (see **[2]**), by a more general family of subsets of $\mathbf{R}^{n+1}_+$. As an application, we look at a family of integral operators (e.g. the Fourier transform) as the action of continuous linear forms, and using the duality established between certain spaces, we obtain pointwise estimates that will allow us to give another proof of well-known bounds for the Fourier transform of H^p functions (see **[4]**, **[12]**). We can also improve the Féjer-Riesz inequality for harmonic extensions (see **[5]**) and we find a generalization considering Hardy spaces defined in terms of arbitrary kernels (see **[14]**). Our main tool will be given by the properties that the tent spaces satisfy (see **[2]**, **[1]**, **[10]**), and in particular their relation with a class of Carleson measures, for which we find a suitable atomic decomposition. We begin by giving some basic definitions.

DEFINITION 1. Let $\Omega = \{\Omega_x\}_{x \in \mathbf{R}^n}$ be a collection of measurable subsets, where $\Omega_x \subset \mathbf{R}^{n+1}_+$. For a measurable function f in $\mathbf{R}^{n+1}_+$ we