PACIFIC JOURNAL OF MATHEMATICS
Vol. 163, No. 2, 1994

A NOTE ON INTERMEDIATE SUBFACTORS

DIETMAR BISCH

In this note we prove that if $N \subset M \subset P$ is an inclusion of II_1 factors with finite Jones index such that $N \subset P$ has finite depth, then $N \subset M$ and $M \subset P$ have finite depth. We show this result by studying the iterated basic constructions for $M \subset P$ and $N \subset P$. In particular our proof gives detailed information about the graphs for $N \subset M$ resp. $M \subset P$. Furthermore, we give an abstract characterization of intermediate subfactors in terms of Jones projections in $N' \cap P_1$, where $N \subset P \subset P_1$ is the basic construction for $N \subset P$ and give examples showing that if $N \subset M$ and $M \subset P$ have finite depth, then $N \subset P$ does not necessarily have finite depth.

1. Introduction. The problem of classifying subfactors of the hyperfinite II_1 factor is one of the most challenging problems in operator algebras. Starting with an inclusion $N \subset M$ of hyperfinite II_1 factors with finite Jones index $[M : N] < \infty$, one constructs the associated Jones tower of factors $N \subset M \subset M_1 \subset M_2 \subset \dots$, where M_{i+1} is the II_1 factor obtained from the *Jones basic construction* for $M_{i-1} \subset M_i$ (see [**Jo1**]). The centralizer algebras $\{M_i' \cap M_j\}_{i \leq j}$ are finite dimensional C^*-algebras sitting in the envelopping II_1 factor $M_\infty = \overline{\bigcup M_k}^w$. Furthermore, inclusions of four such algebras

$$\begin{array}{ccccccc} M_i' & \cap & M_k & \subset & M_i' & \cap & M_{k+1} \\ & \cup & & & & \cup & \\ M_{i+1}' & \cap & M_k & \subset & M_{i+1}' & \cap & M_{k+1} \end{array}$$

satisfy certain symmetry conditions: they form what is called a *commuting square* ([**Po2**], see also [**GHJ**]). All the information contained in this double sequence of finite dimensional algebras is actually contained in the following sequence of commuting squares

$$\begin{array}{cccccccc} M' & \cap & M_k & \subset & M' & \cap & M_{k+1} & \subset \cdots \\ & \cup & & & & \cup & & \\ M_1' & \cap & M_k & \subset & M_1' & \cap & M_{k+1} & \subset \cdots \end{array}$$

which is an invariant for the inclusion $N \subset M$, called the *standard invariant* ([**Po4**] or *paragroup* [**Oc1**]). From this sequence one can form the inclusion $\overline{\bigcup_k M' \cap M_k}^w \subset \overline{\bigcup_k M_1' \cap M_k}^w$ of hyperfinite II_1 von Neumann algebras and ask if these algebras form a model for $N \subset M$,