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CORRECTION TO "TRACE RINGS FOR VERBALLY PRIME ALGEBRAS"

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In **[1]** and **[2]** we incorrectly state a theorem of Razmyslov from **[3]**. We quoted Razmyslov as saying:

For all k and l, $M_{k,l}$ satisfies a trace identity of the form

$$(*)\qquad p(x_1, \dots, x_n, a) = c(x_1, \dots, x_n)\mathrm{tr}(a)$$

where $p(x_1, \dots, x_n, a)$ and $c(x_1, \dots, x_n)$ are central polynomials.

This statement is true if $k \neq l$ and false if $k = l$. We will indicate why this is true and what effect it has on the results of **[1]** and **[2]**. It turns out that **[1]** needs only a very minor comment, but that **[2]** requires a modification to the main theorem and a longer proof in the case of $k = l$.

First, here is a correct version of Razmyslov's theorem:

For all k and l, $M_{k,l}$ satisfies a trace identity of the form

$$(**)\qquad p(x_1, \dots, x_n, a) = \mathrm{tr}(c'(x_1, \dots, x_n))\mathrm{tr}(a)$$

where $p(x_1, \dots, x_n, a)$ is a central polynomial and $c'(x_1, \dots, x_n)$ does not involve any traces.

If $k \neq l$, then the trace of the identity matrix equals $k - l$ which is not zero. So, if we set $a = I$ in $(**)$ we get

$$\mathrm{tr}(c'(x_1, \dots, x_n)) = (k-l)^{-1}p(x_1, \dots, x_n, I).$$

Hence, in this case $\mathrm{tr}(c'(x_1, \dots, x_n))$ equals a central polynomial modulo the identities for $M_{k,l}$, and so $(*)$ is true in this case. To see that $(*)$ is false if $k \neq l$ it is useful to have the following lemma.

LEMMA 1. *Let $f(x_1, \dots, x_n)$ be a pure trace identity for $M_{k,k}$ and write $f(x_1, \dots, x_n) = f_0(x_1, \dots, x_n) + f_1(x_1, \dots, x_n)$, where each monomial in f_0 involves an even number of traces and each monomial in f_1 involves an odd number of traces. Then $f_0(x_1, \dots, x_n)$ and $f_1(x_1, \dots, x_n)$ are each trace identities for $M_{k,k}$.*

Proof. We define an automorphism on $M_{k,k}$. Let $\left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right)$ be an element of $M_{k,k}$, where A, B, C and D are $k \times k$ blocks, and