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EVOLUTIONARY EXISTENCE PROOFS FOR THE PENDANT DROP AND n-DIMENSIONAL CATENARY PROBLEMS

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Two problems about surfaces, both involving a gravitational forcing term, are studied from an evolutionary perspective. It is shown that, in each case, the existence of a unique solution to the associated Boundary Value Problem (BVP) may be established using a suitable mean curvature type flow. By considering two different flows for one of the problems it is illustrated that the best choice of flow, for use in the evolutionary construction of solutions to such mean curvature type BVPs, may often be determined more by geometric considerations than by analytic ones.

0. Basic notation and conventions. Throughout the following let Ω denote an open, bounded domain in $\mathbf{R}^n$, with $C^{2,\alpha}$-boundary $\partial\Omega$, and let A denote the *minimal surface operator*, so that A acts on functions $u \in C^2(\Omega)$ via

$$Au = -D_i(a^i(Du))$$

where the functions $a^i\colon \mathbf{R}^n \to \mathbf{R}$ are defined by

$$a^i(p) = \frac{p_i}{\sqrt{1+|p|^2}}.$$

Here, as in the following, we are using the convention of summing over repeated indices.

Also, for convenience, set $a^i \equiv a^i(Du)$ and $v = \sqrt{1+|Du|^2}$, and introduce the additional notation

$$a^{ij}(p) = \frac{\partial a^i}{\partial p_j} \equiv \frac{1}{\sqrt{1+|p|^2}}\left(\delta_{ij} - \frac{p_i p_j}{1+|p|^2}\right)$$

so that then we may write

$$Au = -a^{ij}D_iD_ju \quad \forall u \in C^2(\Omega)$$

where a^{ij} stands for $a^{ij}(Du)$. Note that the least eigenvalue of the matrix (a^{ij}) is v^{-3}.

Furthermore, for $u \in C^2(\Omega)$, let H denote the mean curvature of the surface graph(u). Observe that the well-known relation $H = -Au$