PACIFIC JOURNAL OF MATHEMATICS
Vol. 164, No. 1, 1994

BRANCHED COVERINGS OF SURFACES WITH AMPLE COTANGENT BUNDLE

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Let $f: X \to Y$ be a branched covering of compact complex surfaces, where the ramification set in X consists of smooth curves meeting with at most normal crossings and Y has ample cotangent bundle. We further assume that f is locally of form $(u, v) \to (u^n, v^m)$. We characterize ampleness of T^*X. A class of examples of such X, which are branched covers of degree two, is provided.

1. Introduction. An interesting problem in surface theory is the construction and characterization of surfaces with ample cotangent bundle. They are necessarily algebraic surfaces of general type. Natural examples occur among the complete intersection surfaces of abelian varieties. More subtle examples are those constructed by Hirzebruch **[6]** using line-arrangements in the plane. The characterization of those of Hirzebruch's line-arrangement surfaces with ample cotangent bundle is due to Sommese **[8]**. In this article, we will give a characterization of ampleness of the cotangent bundle of a class of surfaces which branch cover another surface with ample cotangent bundle. We will also construct certain branched coverings of explicit line-arrangement surfaces; these constructions will again have ample cotangent bundle.

For any vector bundle E over a base manifold M, the projectivization $\mathbf{P}(E)$ is a fiber bundle over M, with fiber $\mathbf{P}_q(E)$ over $q \in M$ given by $\mathbf{P}_q(E) \approx (E_q^* \backslash 0)/\mathbf{C}^*$. There is a tautological linebundle ξ_E over $\mathbf{P}(E)$ satisfying (i) $\xi_E|_{\mathbf{P}_q(E)} \approx O(1)_{\mathbf{P}_q(E)}\ \forall q \in M$, and (ii) the projection $\rho_E: \mathbf{P}(E) \to M$ gives $\rho_{E_*}(\xi_E) \approx E$. In the case that $E = T^*X$ we will denote $\rho_E = \rho_{T^*X}$ simply by ρ.

DEFINITION. The vector bundle E is *ample* if ξ_E over $\mathbf{P}(E)$ is ample.

In §2 we prove preliminary results along with:

THEOREM 1.1. *Let X and Y be compact complex surfaces, with Y having ample cotangent bundle. Let $f: X \to Y$ be a branched covering which can be locally represented with coordinate charts of form $f: (u, v) \to (u^n, v^m)$. Let f have ramification set $\bigcup B_j$ in X*