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PERIODIC POINTS ON NILMANIFOLDS AND SOLVMANIFOLDS

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Let M be a compact manifold and $f: M \to M$ a self map on M. For any natural number n, the nth iterate of f is the n-fold composition $f^n: M \to M$. The fixed point set of f is $\text{fix}(f) = \{x \in M: f(x) = x\}$. We say that $x \in M$ is a periodic point of f if x is a fixed point of some f^n and we denote the set of all periodic points of f by $\text{per}(f) = \bigcup_{n=1}^{\infty} \text{fix}(f^n)$. A periodic point x of f is said to have minimal period k provided that k is the smallest integer for which $x \in \text{fix}(f^k)$. We say that $\text{per}(f)$ is *homotopically finite*, denoted $\text{per}(f) \sim$ finite, iff there is a g homotopic to f such that $\text{per}(g)$ is finite. When M is a torus B. Halpern has shown that $\text{per}(f) \sim$ finite iff the sequence of Nielsen numbers $\{N(f^n)\}_{n=1}^{\infty}$ is bounded. The main objective of this work is to extend these results to all nilmanifolds and to consider to what extent they can be extended for compact solvmanifolds. A compact nilmanifold is a coset space of the form $M = G/\Gamma$ where G is a connected, simply connected nilpotent Lie group and Γ is a discrete torsion free uniform subgroup. For these spaces we have the additional result that when the homotopy can be accomplished, the resulting g satisfies $|\text{fix}(g^n)| = N(f^n)$ for all n with $N(f^n) \neq 0$ and if $\{N(f^n)\}_{n=1}^{\infty} = \{0\}$ then we can choose g to be periodic point free. Also, when f is induced by a homomorphism $F: G \to G$, then we can write $g = ufv$ where u and v are isotopic to the identity. This form for the homotopy is used to find sufficient conditions for $\text{per}(f) \sim$ finite when M is a solvmanifold. We then present a model for a specific class of solvmanifolds where these conditions can be considered. This allows us to prove the general result in a variety of low dimensional examples.

Consider two self maps f and g on M. Classical Nielsen fixed point theory concerns itself with the study of how $\text{fix}(f)$ and $\text{fix}(g)$ are related when g is homotopic to f, written $g \sim f$. In particular, the Nielsen number $N(f)$ of f is a lower bound on the number of fixed points of any map in the homotopy class of f. The Nielsen number $N(f)$ is defined by partitioning $\text{fix}(f)$ into equivalences classes (called Nielsen classes) and assigning an integer index to each class. Points $x, y \in \text{fix}(f)$ are said to be Nielsen equivalent, denoted $x \sim_f y$ provided that there is a path ω from x to y in which (rel endpoints) $\omega \sim f\omega$. For a smooth map and isolated fixed points the fixed point