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UNIT INDICES OF SOME IMAGINARY COMPOSITE QUADRATIC FIELDS

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Let K be an imaginary abelian number field of type $(2, 2, 2, 2)$ not containing the 8th cyclotomic field. Using the fundamental units of real quadratic subfields of K, we give a necessary and sufficient condition for the unit index Q_K of K to be equal to 2.

1. Introduction and results. Let K be an imaginary abelian number field and K_0 the maximal real subfield of K. Let E and E_0 be the groups of units of K and K_0, respectively, and let W be the group of roots of unity in K. Then we call the group index

$$Q_K = [E : WE_0]$$

the unit index of K.

Using the character group of K, H. Hasse [**2**] gave sufficient conditions for Q_K to be equal to 1 or 2, by which we can determine Q_K for some types of fields K. However by his method we cannot always determine Q_K for arbitrary K, even if K is an imaginary composite quadratic field. (We call a field K a composite quadratic field if K is a composite of quadratic fields.) K. Yoshino and the author [**3**, **4**] gave criteria to determine Q_K of K with Galois group $\mathrm{Gal}(K/\mathbf{Q})$ of type $(2, 2)$ and $(2, 2, 2)$.

In this paper we extend our previous results [**3**, **4**] to the case that K has Galois group $\mathrm{Gal}(K/\mathbf{Q})$ of type $(2, 2, 2, 2)$ and does not contain the 8th cyclotomic field, and then, we give a necessary and sufficient condition for the unit index Q_K to be equal to 2.

NOTATION. $\mathbf{N}, \mathbf{Z}, \mathbf{Q}$: the sets of natural numbers, rational integers and rational numbers, respectively,

$\underset{2}{=}$: the equality except rational quadratic factors,

$d_0, d_1, d_2, \ldots, d_7$: square-free positive integers such that $d_4 \underset{2}{=} d_2 d_3$, $d_5 \underset{2}{=} d_3 d_1$, $d_6 \underset{2}{=} d_1 d_2$, $d_7 \underset{2}{=} d_1 d_2 d_3$ and $d_0 \neq d_i$ $(i = 1, 2, \ldots, 7)$,

$K = \mathbf{Q}(\sqrt{-d_0}, \sqrt{d_1}, \sqrt{d_2}, \sqrt{d_3})$: an imaginary composite quadratic field of degree 16,