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# THE HYPERSPACES OF INFINITE-DIMENSIONAL COMPACTA FOR COVERING AND COHOMOLOGICAL DIMENSION ARE HOMEOMORPHIC

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**A notion of true dimension theory is defined to which is assigned a dimension function $D$. We consider those $D$ which have an enhanced Bockstein basis; these include $D = \dim$ and $D = \dim_G$ for any abelian group $G$. We prove that for each countable polyhedron $K$, the set of compacta $X \in 2^{\mathbf{Q}}$ with $K \in \mathrm{AE}(\{X\})$ is a $G_\delta$-subspace. We apply this fact to show that the hyperspace of the Hilbert cube Q consisting of compacta (or continua) $X$ with $D(X) \leq n$ is a $G_\delta$-subspace. Let $D^{\geq n}$ (resp., $D^{\geq n} \cap C(\mathbf{Q})$ ) denote the space of compacta $X$ (resp., continua) with $D(X) \geq n$. We prove that $\{D^{\geq n}\}_{n=1}^{\infty}$ and $\{D^{\geq n} \cap C(\mathbf{Q})\}_{n=2}^{\infty}$ are absorbing sequences for $\sigma$-compact spaces. This yields that each $D^{\geq n}$ and $D^{\geq n+1} \cap C(\mathbf{Q})$ ( $n \geq 1$ ) is homeomorphic to the pseudoboundary $B$ of Q; their respective complements are homeomorphic to the pseudointerior of Q; and the intersections $\bigcap_n D^{\geq n}$, $\bigcap_n D^{\geq n} \cap C(\mathbf{Q})$ are homeomorphic to $B^\infty$, the absorbing set for the class of $F_{\sigma\delta}$-sets. Results for the hyperspaces of compacta $X$ for which $D(X) \geq n$ uniformly are also obtained.**

**1. Introduction.** The ultimate objective of this paper is topological identification of certain hyperspaces of compacta related to dimension. Hyperspaces we are dealing with are subspaces of $2^{\mathbf{Q}}$, the hyperspace of all compacta of the Hilbert cube $\mathbf{Q} = [0, 1]^\infty$ endowed with the Vietoris topology (Hausdorff metric) or $C(\mathbf{Q})$, the hyperspace of continua. By dimension we mean true dimension (§2) which includes covering dimension dim and cohomological dimension $\dim_G$ for any abelian group $G$. The classical result, IV.45.4 of **[Kur]**, states that the hyperspace of compacta with $\dim \leq n$ is an absolute $G_\delta$-set (equivalently: a $G_\delta$-subset of $2^{\mathbf{Q}}$ ). A natural question arises whether the above statement holds true for $\dim_G$. (If this were not true for $G = \mathbb{Z}$, then it would implicitly yield examples of Dranishnikov type **[Dr1]**.) The question for, $G = \mathbb{Z}$, was brought to our attention in January 1991 by R. Pol. Apparently this had already been considered by the authors of **[DvMM]**. Furthermore, Robert Cauty has communicated to us a proof that this set is coanalytic. We shall prove (Corollary