PACIFIC JOURNAL OF MATHEMATICS
Vol. 164, No. 1, 1994

ENTROPY VERSUS ORBIT EQUIVALENCE FOR MINIMAL HOMEOMORPHISMS

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Every minimal homeomorphism of a Cantor set is strongly orbit equivalent to a homeomorphism of zero entropy. The dyadic adding machine is strongly orbit equivalent to homeomorphisms of all entropies.

1. Introduction and definitions. We construct, using the lexicographic or "adic" maps of Vershik (as modified by Herman, Putnam, and Skau [P], [HPS], [Sk]), examples of minimal homeomorphisms of all possible (topological) entropies (including infinite) which are "strongly" orbit equivalent to the dyadic adding machine. In particular, this answers in a decisively negative way the question as to whether orbit equivalent homeomorphisms have the same entropy. We also show that any minimal homeomorphism of the Cantor set is strongly orbit equivalent to one of zero entropy.

We recall some definitions. A source for these is the survey article of Skau [Sk]. A Bratteli diagram B is a directed graph whose vertex set decomposes into finite subsets, "levels" or "rows", B_k $(k = 0, 1, \ldots)$, together with edges from vertices in B_k to vertices in B_{k+1}; additionally, every vertex of B_k is joined to a vertex of B_{k+1}. The Bratteli diagram is *simple* if for all k, there exists $k' > k$ such that for every vertex v in B_k and every vertex v' in $B_{k'}$, there is a path from v to v'. The Bratteli diagram is *pointed* if $|B_0| = 1$, that is, there is a distinguished top vertex. The set of infinite paths, usually denoted X, is called the *Bratteli compactum*, and is a compact zero dimensional metric space, i.e., a compact Boolean space.

One can describe a Bratteli diagram using rectangular matrices with nonnegative integer entries. If $|B_k| = w(k)$ (with $w(0) = 1$), define a matrix M_k in $\mathbf{Z}^{w(k+1)\times w(k)}$ as follows. Index the vertices in B_k by an initial segment of the positive integers, and set the i, j entry to be the number of edges from point j of B_k to point i of B_{k+1}. One may then form the limit *dimension group*, the direct limit, as ordered abelian groups,

$$(1.1) \qquad G_B = \lim \mathbf{Z}^{w(0)} \xrightarrow{M_0} \mathbf{Z}^{w(1)} \xrightarrow{M_1} \mathbf{Z}^{w(2)} \xrightarrow{M_2} \cdots .$$