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THE JONES POLYNOMIAL OF PARALLELS AND APPLICATIONS TO CROSSING NUMBER

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In this paper we study the Jones polynomial of the parallels of a link or knot. From the extremal exponents occurring we derive lower bounds on the crossing number of the knot, reproducing in particular a number of results of Thistlethwaite. We apply these techniques to give lower bound on the crossing number of some simple satellites of adequate and semi-adequate knots (including cable satellites) that are usually quadratic in the degree of the satellite.

Introduction. Several recent papers have used the Jones polynomial to study the crossing number of a link. First, Kauffman [**2**], using the Jones polynomial, showed that any two reduced, connected, alternating diagrams for a link have the same crossing number. This result was extended independently by Murasugi [**5**] and Thistlethwaite [**6**] showing that a reduced alternating diagram has the minimal crossing number. Thistlethwaite [7, **8**] has extended these results, using the Kauffman (or semi-oriented) polynomial, to show that the writhe of a reduced alternating diagram of an alternating link is an isotopy invariant of L and to show that an adequate diagram of a link, has minimal crossing number.

In this paper, we will reproduce these results and some other results of Thistlethwaite using instead the Jones polynomial of the parallels of a link. Using this method, we will be able to give lower bounds for the crossing number of the r-fold parallels of an adequate knot, which in most cases are quadratic in r. We will further show that these lower bounds are stable under a class of variations. These variants may be thought of as being the satellites coming from flows that are C^1-close to the parallel flow, in the sense of [**1**].

The Kauffman bracket polynomial of a planar diagram of an unoriented link is an element $\langle D\rangle \in \mathbf{Z}[A, A^{-1}]$ defined by the following procedure. A state for D is defined to be a map s from the crossings of D (which we take to be indexed by $1 \leq i \leq n$) to $\{-1, 1\}$. Let sD denote the diagram obtained from D by nullifying the crossings of D according to s as in Figure 1. For any s, sD consists of disjoint simple closed curves. Let $|sD|$ denote the number of such simple