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ORIENTED ORBIFOLD COBORDISM

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A complete set of invariants (generalized Pontrjagin numbers) for rational oriented orbifold cobordism is determined. Using these numbers we prove that for any odd dimensional oriented orbifold Q there is a nonzero multiple of Q which bounds another orbifold and that, unlike the manifold case, this need not be true for $4k+2$ dimensional orbifolds. In addition we construct generators for the rational orbifold cobordism ring and show that it is a free commutative ring on these.

Introduction. This paper establishes a foundation for oriented orbifold cobordism in a manner analogous to Thom's results for oriented manifolds. Thom's main theorem states that some multiple of an oriented compact manifold bounds another such manifold if and only if its Pontrjagin numbers are zero. Orbifolds are like manifolds, except that they locally look like $\mathbf{R}^n/G$ for G a finite group. For each H a finite subgroup of $\mathrm{SO}(n)$ we define H characteristic numbers of the compact oriented orbifold Q which account for its H singular set and are its Pontrjagin numbers when H is trivial. Our main theorem is that some multiple of Q bounds another compact oriented orbifold if and only if all its H characteristic numbers are zero for all finite subgroups H of $\mathrm{SO}(n)$.

Thom's corollary that every manifold with dimension not divisible by four rationally bounds does not translate exactly in the orbifold case. There is no reason to expect it to as the H singular set of even an odd dimensional orbifold may have nonzero Pontrjagin numbers. However, a careful consideration of orientations and the twisted cohomology of the classifying space of the centralizer of H, $C_{\mathrm{O}(d)}(H)$, shows that any odd dimensional orbifold rationally bounds another orbifold W with the same set of local groups. The result for $(4k+2)$-dimensional orbifolds is not as neat. We can isolate a certain class of finite subgroups of $\mathrm{SO}(4k+2)$, which includes all finite abelian subgroups, such that any $(4k+2)$-dimensional orbifold with all local groups in this class rationally bounds. This set does not include all finite subgroups as we construct a seventy-dimensional orbifold with nonzero characteristic numbers which therefore does not rationally bound.