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ONE-PARAMETER FIXED POINT INDICES

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Let $F: X \times I \to X$ be a PL homotopy, where X is a compact connected PL n-dimensional manifold, in the euclidean space $\mathbb{R}^n$, $n \geq 4$, and let $P: X \times I \to X$ be the projection. A fixed point of F is a point $(x, t) \in X \times I$ such that $F(x, t) = x$. The set of all the fixed points of F is denoted by $\mathrm{Fix}(F)$. For a family V of isolated circles of fixed points of F we define two indices: $\mathrm{ind}_1(F, V)$—which is an element in the first homology group $H_1(E)$, where E is the space of paths in $X \times I \times X$ from the graph of F to the graph of P; and $\mathrm{ind}_2(F, V)$—which is an element in the group $\mathbb{Z}_2$ with two elements. We prove that there is a compact neighborhood N of V and a homotopy from F to H $\mathrm{rel}\, X \times I \backslash N$ such that $\mathrm{Fix}(H) = \mathrm{Fix}(F) \backslash V$ if and only if $\mathrm{ind}_1(V, F) = 0$ and $\mathrm{ind}_2(V, F) = 0$. The indices $\mathrm{ind}_1(V, f)$ and $\mathrm{ind}_2(V, F)$ are defined via the degrees, $\deg_1(g)$ and $\deg_2(g)$, for maps $g: S^1 \times S^m \to S^m$. Moreover, we show how to modify F to create circles of fixed points with prescribed indices.

Introduction. In this paper we define two indices for fixed points of homotopies between two selfmaps of a manifold, and then show that these indices provide us with sufficient and necessary conditions for removing some or all of the fixed point set, in a controlled manner. Let $F: X \times I \to X$ be a PL homotopy, where X is a compact connected PL n-dimensional manifold, contained in the euclidean space $\mathbb{R}^n$, let $n \geq 4$, and let $P: X \times I \to X$ be the projection. A fixed point of F is a point $(x, t) \in X \times I$ such that $F(x, t) = x$. The set of all the fixed points of F is denoted by $\mathrm{Fix}(F)$. In this setting, isolated circles of fixed points are the generic form of fixed points, as isolated individual fixed points are in the classical setting. The two indices, $\mathrm{ind}_1(F, V)$ and $\mathrm{ind}_2(F, V)$, are defined for a family V of finitely many isolated circles of fixed points of F. The first index, $\mathrm{ind}_1(F, V)$, is an element in the first homology group $H_1(E)$, where E is the space of paths in $X \times I \times X$ from the graph of F to the graph of P, and is a slight generalization of the first obstruction discussed in [DG]. It is mentioned in [DG] that a solution to the one parameter fixed point problem in the transverse case can be found in [HQ], via an obstruction lying in the 1-dimensional framed bordism group of