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GENERIC 8-DIMENSIONAL ALGEBRAS WITH MIXED BASIS-GRAPH

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Deformation theory is the appropriate tool for describing the irreducible components of the scheme Alg_n which parametrizes the structures of n-dimensional associative algebras with unit. Each component is "dominated" by one generic or quasi-generic algebra or family of algebras (genericity means that the algebra or the family has only trivial infinitesimal deformations, and quasi-genericity means that the algebra or the family has non trivial infinitesimal deformations, but no algebraic deformation). The components dominated by a generic algebra (or family) are reduced, while the components dominated by a quasi-generic family are non reduced. The invariants we use for that classification are the basis-graph, both weighted and unweighted, of an associative algebra. In this paper, we classify the 8-dimensional algebras with mixed basis-graph and give lower bounds for the numbers of irreducible components of the scheme Alg_8, reduced and non reduced.

I. Introduction. This paper is a new contribution to the question treated in previous works ([**Ha**], [**Ma**], [**DP1**], [**DP2**]), namely the study of the irreducible components of the scheme $\mathbf{Alg}_n$ which parametrizes the structures of n-dimensional associative algebras with unit. The importance of this question was put forward by Gabriel (see [**Ga**]). In [**DP2**], the author proved some general lemmas, which enable us to construct deformations of n-dimensional algebras from deformations in dimension less than n. As already known from [**Ha**] and [**DP2**], the main tool is M. Gerstenhaber's theory of deformations of algebras, using Hochschild cohomology (cf. [**Ge**]) and the appropriate invariants for the classification work are the basis-graph and the weighted basis-graph, as defined in [**Sc1**] by M. Schaps. In dimension equal to or greater than 6, the task of writing down a complete deformation chart is unilluminating, as the number of different isomorphism classes of algebras increases very fast, but it is still possible to determine good lower bounds for the number $p(n)$ of reduced irreducible components and the total number $q(n)$ of irreducible components of $\mathbf{Alg}_n$, including the non reduced ones.

In §II, we recall the main definitions and theorems, without proof, from the theory of deformations of associative unitary algebras; the