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FREE PRODUCTS OF COMBINATORIAL STRICT INVERSE SEMIGROUPS

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Each combinatorial strict inverse semigroup S is determined by (1) a partially ordered set X which in fact is the partially ordered set of the $\mathscr{J}$-classes of S, (2) pairwise disjoint sets I_α indexed by the elements of X which in fact form the collection of $\mathscr{D}$- (equivalently: $\mathscr{J}$-) related idempotents and (3) structure mappings $f_{\alpha,\beta}: I_\alpha \to I_\beta$ for $\alpha \geq \beta$ satisfying certain compatibility conditions. The multiplication on S can be described in terms of the parameters X, I_α, $f_{\alpha,\beta}$. Conversely, the system $(X; I_\alpha, f_{\alpha,\beta})$ can be characterized abstractly in order that it defines a uniquely determined combinatorial strict inverse semigroup. In this paper, the constituting parameters X, I_α, $f_{\alpha,\beta}$ of the combinatorial strict inverse free product S of a collection of combinatorial strict inverse semigroups S_i are described in terms of the parameters of the semigroups S_i.

As an application it is shown that the word problem for such a free product in general is not decidable.

1. Introduction. The ($\mathscr{V}$-)free product of an arbitrary family $\{S_i | i \in I\}$ of algebras of the same type all of them belonging to the class $\mathscr{V}$ is the coproduct $\prod^* S_i$ in $\mathscr{V}$. There are homomorphisms $\phi_i: S_i \to \prod^* S_i$, $i \in I$, and for any $T \in \mathscr{V}$ and homomorphisms $\psi_i: S_i \to T$, $i \in I$, there is a unique homomorphism $\psi: \prod^* S_i \to T$ such that $\phi_i \psi = \psi_i$ for all $i \in I$.

From purely universal algebraic considerations it follows that the free product exists for any variety $\mathscr{V}$ of inverse semigroups and is generated by isomorphic copies of the members of the given family (see, for instance, Grätzer [5]). Free products have been studied for several classes of semigroups. Semilattice free products and semilattice of groups free products are considered in the book of Petrich [14]. Band, completely simple and completely regular free products have been investigated by Jones [9, 6, 11]. Inverse semigroup free products have been studied by Jones [7, 8, 10] and Jones, Margolis, Meakin and Stephen [12]. The aim of this paper is to describe *combinatorial strict inverse semigroup* free products. A combinatorial strict

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