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INTERPOLATION SUBMANIFOLDS OF THE UNITARY GROUP

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An interpolation subset in the boundary of a domain is a closed set in which every continuous (or smooth) function can be extended as a holomorphic function inside the domain and continuous (or smooth, respectively) up to the boundary. In this paper we give some geometric description for submanifolds in the unitary group to be interpolation sets for the domain obtained by taking polynomial hull of the unitary group. In particular, we retrieved corresponding results on the polydisc.

The goal of this paper is to characterize the interpolation manifolds in the unitary group $\mathrm{U}(n)$, which is regarded as an n^2-dimensional real analytic submanifold in $C^{n^2} = R^{2n^2}$. The theme of this topic started from the work by Henkin and Tumanov in **[5]**, and Burns-Stout in **[2]** who proved the case for real analytic interpolation manifolds on the boundaries of some pseudoconvex domains. Then a lengthy cycle of works (Hakim-Sibony **[4]**, Henkin-Tumanov **[5]**, Stout **[13]**, etc.) followed which mainly study the case for interpolation manifolds in the boundaries of strongly pseudoconvex domains. By using an embedding technique, Saeren's paper **[11]** was the first to deal with the case for the interpolation manifolds in the polydisc. In what follows, we shall deal with similar problems for $\mathrm{U}(n)$, which contains the polydisc as an n-dimensional real analytic submanifold.

The paper is organized in the following way.

In §1, we give some basic definitions and properties related to the unitary group $\mathrm{U}(n)$. In particular, the polynomial hull of $\mathrm{U}(n)$ is described. After introducing the open-cone condition and the closed-cone condition, which bear some resemblance to the polydisc case, §2 contains the statement of all the results in this paper. Section 3 provides mainly the technical details for the proof of the results stated in §2. Finally, §4 contains some remarks that relate our work to that of Jimbo-Sakai **[7]** and that of Saerens **[11]**, **[12]**.

1. Definitions and certain properties of $\mathrm{U}(n)$. In this section, we are going to introduce several notations and definitions that we will