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PERTURBATIONS OF CERTAIN REFLEXIVE ALGEBRAS

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In this note we use cohomological techniques to prove that if there is a linear map between two CSL algebras which is close to the identity, then the two CSL algebras are similar. We use our result to show that if $\mathscr{L}$ is a purely atomic, hyperreflexive CSL with uniform infinite multiplicity which satisfies the 4-cycle interpolation condition, then there are constants δ, $C > 0$ such that whenever $\mathscr{M}$ is another CSL such that $d(\operatorname{Alg}\mathscr{L}, \operatorname{Alg}\mathscr{M}) < \delta$, then there is an invertible operator S such that $S \operatorname{Alg}\mathscr{L} S^{-1} = \operatorname{Alg}\mathscr{M}$ and $\|S\|\,\|S^{-1}\| < 1 + Cd(\operatorname{Alg}\mathscr{L}, \operatorname{Alg}\mathscr{M})$.

1. Introduction and preliminaries. In this paper, we consider two related types of perturbation questions for CSL algebras. The first deals with the problem of perturbing a linear isomorphism between two algebras to obtain an algebra isomorphism, and the second deals with the problem of deciding whether, for two such algebras, closeness implies isomorphism.

Perturbation questions of this kind were considered by Kadison and Kastler for von Neumann algebras in **[18]** and further studied by many authors, including Christensen (**[3, 4, 5]**). Johnson (**[16]**) and Raeburn and Taylor (**[21]**) obtained results concerning perturbations of closed subalgebras of Banach algebras. Their results show that if $\mathscr{A}$ is a closed subalgebra of a Banach algebra $\mathscr{B}$ and certain cohomology groups for $\mathscr{A}$ vanish, then any closed subalgebra of $\mathscr{B}$ "sufficiently close" to $\mathscr{A}$ is actually isomorphic to $\mathscr{A}$. The nonselfadjoint case was considered first for nest algebras by Lance in **[19]**. Perturbations of other nonselfadjoint operator algebras were considered by Choi and Davidson (**[2]**), Davidson (**[6]**).

In §2, we prove Theorem 3 which shows that if two CSL algebras $\mathscr{A}_1$ and $\mathscr{A}_2$ are linearly isomorphic via an isomorphism close to the identity, they they are actually spatially isomorphic via an isomorphism which is close to a unitary equivalence. In §3, we introduce the 4-cycle interpolation property, which is closely related to a lattice condition appearing in **[12]** and to the notion of interpolating lattice introduced in **[7]**. The main result of §3, Theorem 16, shows that if $\mathscr{A}_1$ is a CSL algebra which is sufficiently close to a purely atomic,