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COMPLETE OPEN MANIFOLDS OF NON-NEGATIVE RADIAL CURVATURE

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We generalize the Toponogov hinge theorem and the Alexandrov convexity to the context of radial curvature, and study complete open Riemannian manifolds of non-negative radial curvature.

0. Introduction. It is well-known that a non-negative curved manifold has some interesting characters as exemplified in the Soul theorem ([CG]) or the Toponogov splitting theorem ([T]). In such theorems, Toponogov's comparison theorem plays an essential role.

Throughout this paper let M be a connected complete Riemannian manifold of dimension $n \geq 2$. For a point $o \in M$, the sectional curvature K_M of M restricted to those planes that are tangent to some minimal geodesic starting from o is called minimal radial curvature from o and is denoted by $K_o^{\min}$. The notion of radial curvature was initiated by Klingenberg in [K] to prove a homotopy sphere theorem for compact simply-connected manifolds with $\frac{1}{4}$-pinched radial curvature. Also in the case where M is noncompact and o is a pole of M, Greene and Wu have shown some results related to the radial curvature from o (see [GW]).

In [M], it is shown that Toponogov's comparison theorem holds for the edge angles at x_1 and x_2 of a minimal geodesic triangle with vertices at o, x_1, and x_2 under suitable condition on $K_o^{\min}$. Moreover by using this fact, some results related to the radial curvature from o were obtained in [M] or [MS]. For example,

THEOREM 0.1 (*Theorem* A *in* [MS]). *A complete noncompact Riemannian manifold M which contains a point o such that $K_o^{\min} > 0$ has exactly one end.*

THEOREM 0.2 (*Theorem* C *in* [MS]). *Let M be noncompact with a point o such that $K_o^{\min} \geq 0$. If*

$$\lim_{r\to\infty} \frac{\mathrm{vol}B(o,r)}{b_0(r)} > \frac{1}{2},$$