PACIFIC JOURNAL OF MATHEMATICS
Vol. 165, No. 1, 1994

q-CANONICAL COMMUTATION RELATIONS AND STABILITY OF THE CUNTZ ALGEBRA

P. E. T. JØRGENSEN, L. M. SCHMITT AND R. F. WERNER

We consider the q-deformed canonical commutation relations $a_i a_j^* - q a_j^* a_i = \delta_{ij}\mathbb{1}$, $i, j = 1, \dots, d$, where d is an integer, and $-1 < q < 1$. We show the existence of a universal solution of these relations, realized in a C^*-algebra $\mathscr{E}^q$ with the property that every other realization of the relations by bounded operators is a homomorphic image of the universal one. For $q = 0$ this algebra is the Cuntz algebra extended by an ideal isomorphic to the compact operators, also known as the Cuntz-Toeplitz algebra. We show that for a general class of commutation relations of the form $a_i a_j^* = \Gamma_{ij}(a_1, \dots, a_d)$ with Γ an invertible matrix the algebra of the universal solution exists and is equal to the Cuntz-Toeplitz algebra. For the particular case of the q-canonical commutation relations this result applies for $|q| < \sqrt{2} - 1$. Hence for these values $\mathscr{E}^q$ is isomorphic to $\mathscr{E}^0$. The example $a_i a_j^* - q a_i^* a_j = \delta_{ij}\mathbb{1}$ is also treated in detail.

1. Introduction. In this paper we study the relations

$$a_i a_j^* - q a_j^* a_i = \delta_{ij}\mathbb{1}, \qquad i, j = 1, \dots, d, \tag{1}$$

for bounded operators a_i, $i = 1, \dots, d$, $d < \infty$ on a Hilbert space, and a deformation parameter q satisfying $-1 < q < 1$. For $q = 0$ these relations are known as the Cuntz relations, and it is well known that in this case the C^*-algebra generated by the a_i is essentially unique: it is either the so-called Cuntz-Toeplitz algebra, or the quotient of this algebra by its only closed two-sided ideal (generated by $\mathbb{1} - \sum_i a_i^* a_i$), which is known as the Cuntz algebra $\mathscr{O}_d$. Our main result in this paper is that the same statement holds for the relations (1) for small q. The technique we use also applies to more general relations of the form

$$a_i a_j^* = \Gamma_{ij}(a_1, \dots, a_d), \tag{2}$$

where Γ is an invertible matrix of functions in the functional calculus of d variables, which satisfy a continuity and a growth condition specified below. We then show that one can decompose the generators as $a_i = v_i \rho$, where the v_i satisfy the Cuntz relations, and ρ satisfies