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STRONGLY APPROXIMATELY TRANSITIVE GROUP ACTIONS, THE CHOQUET-DENY THEOREM, AND POLYNOMIAL GROWTH

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Let G be a group. A Borel G-space $\mathscr{X}$ with a σ-finite quasi-invariant measure α is called strongly approximately transitive (SAT) if there exists an absolutely continuous probability measure ν such that the closed convex hull $\operatorname{co}(G\nu)$ of the orbit $G\nu$ coincides with the space of absolutely continuous probability measures on $\mathscr{X}$. Call a G-space $(\mathscr{X}, \alpha)$ purely atomic if α is purely atomic. Boundaries of stationary random walks on countable G are always SAT and provide many examples of nonatomic SAT actions. The class of nonatomic SAT G-spaces also includes certain homogeneous spaces of locally compact groups. Every countable nonamenable group and also some amenable groups admit nonatomic SAT actions. However, if G contains a countable nilpotent subgroup of finite index then every SAT G-space is necessarily purely atomic. This implies the Choquet-Deny theorem for such groups. Existence of nonatomic SAT actions is related to growth conditions. A finitely generated solvable group has polynomial growth if and only if it does not admit nonatomic SAT actions.

1. Introduction. Let G be a group and $\mathscr{X}$ a Borel G-space with a σ-finite quasi-invariant measure α. We shall denote by $L^1(\mathscr{X}, \alpha)$ the space of complex measures absolutely continuous with respect to α and by $L^1_1(\mathscr{X}, \alpha) \subseteq L^1(\mathscr{X}, \alpha)$ the subspace of probability measures. For $g \in G$ and $\mu \in L^1_1(\mathscr{X}, \alpha)$ we shall write $g\mu$ for the measure $(g\mu)(A) = \mu(g^{-1}A)$. The action of G on $(\mathscr{X}, \alpha)$ is called *approximately transitive* (AT) if for every pair $\nu_1, \nu_2 \in L^1_1(\mathscr{X}, \alpha)$ and every $\varepsilon > 0$ there exists $\nu \in L^1_1(\mathscr{X}, \alpha)$ such that the total variation norm distances between ν_i, $i = 1, 2$, and the convex hull $\operatorname{co}(G\nu)$ of the orbit $G\nu$ are both less than ε. The concept of approximate transitivity was introduced by Connes and Woods [1] to provide a necessary and sufficient condition for an approximately finite dimensional von Neumann factor to be ITPFI (AT characterizes the flow of weights of ITPFI factors). Connes and Woods also observed [2] that the group action on the Poisson boundary of a (not necessarily stationary) random walk on a locally compact second countable group is approximately transitive. They showed that in the case $G = \mathbb{R}$