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R-GROUPS AND ELLIPTIC REPRESENTATIONS FOR SL_n

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To Elizabeth

We determine the reducibility and number of components of any representation of $\mathrm{SL}_n(F)$ which is parabolically induced from a discrete series representation. The R-groups are computed in terms of restriction from $\mathrm{GL}_n(F)$, extending the results of Gelbart and Knapp. This yields an explicit description of the elliptic tempered representations of $\mathrm{SL}_n(F)$. We also describe those tempered representations which are not irreducibly induced from elliptic representations.

Introduction. We continue our investigation of those representations of classical p-adic groups which are parabolically induced from the discrete series. We now consider the group $G = \mathrm{SL}_n(F)$. We will describe explicit criteria for reducibility of induced representations, determine the number of constituents of such representations, and develop criteria for the constituents to be elliptic. Moreover, we can describe those irreducible tempered restrictions of G which are not elliptic, and are also not irreducibly induced from an elliptic representation.

We use the technique of restriction from $\widetilde{G} = \mathrm{GL}_n(F)$. This technique has been used by several authors to describe various aspects of the representation theory of G [**4, 5, 6, 7, 14, 19, 20, 21, 22, 24, 30**]. Our purpose here is to use some of these results to obtain information on the structure of the generalized principal series for G.

Let $P = MN$ be a parabolic group of G. Suppose that σ is an irreducible discrete series representation of M. We wish to determine when the unitarily induced representation $i_{G,M}(\sigma)$ is reducible, and if so, what is the structure of its components. We use the theory of R-groups, as developed by Knapp and Stein [**18**], and Silberger [**28**]. This, along with the multiplicity one result of Howe and Silberger [**14**], determines the structure of the commuting algebra $C(\sigma)$.

The R-group is a quotient of the subgroup $W(\sigma)$ of Weyl group elements which fix σ. If Δ' is the collection of reduced roots for which the Plancherel measure of σ vanishes, then $R \simeq W(\sigma)/W'$, where W' is the group generated by reflections in the roots in Δ'. For the groups $\mathrm{Sp}_{2n}(F)$, $\mathrm{SO}_n(F)$, and $\mathrm{U}_n(F)$, we were able to explicitly