PACIFIC JOURNAL OF MATHEMATICS
Vol. 165, No. 1, 1994

ON CLOSED HYPERSURFACES OF CONSTANT SCALAR CURVATURES AND MEAN CURVATURES IN S^{n+1}

SHAOPING CHANG

We consider in this note the following question: given a closed Riemann n-manifold of constant scalar curvature, how can it be minimally immersed in the round $(n+1)$-sphere? Our main result states that the immersion has to be isoparametric if the number of its distinct principal curvatures is three identically. This provides another piece of supporting evidence to a conjecture of Chern.

0. Introduction. Consider $\mathscr{M}^n_{\text{closed}}$ the set of all the closed minimal hypersurfaces of constant scalar curvatures R in the unit round $(n+1)$-sphere S^{n+1}. Let $\mathscr{R}_n \subset \mathbf{R}$ be the collection of all the possible values of such R's. Chern [12] posed the following:

Chern Conjecture. For any $n \geq 3$, $\mathscr{R}_n$ is a discrete subset of the real numbers.

This is a very interesting conjecture in the theory of minimal submanifolds in spheres. To attack this problem, it will be most helpful if one has a good guess on what $\mathscr{M}^n_{\text{closed}}$ is for each n. When $n=3$, from his work on the exterior differential systems R. Bryant [1] proposed the following:

Bryant Conjecture. A piece of minimal hypersurface of constant scalar curvature in S^4 is isoparametric of type $g \leq 3$.

Here a hypersurface (not necessarily compact) M^n in S^{n+1} is said to be *isoparametric of type* g if it has constant principal curvatures $\lambda_1 < \cdots < \lambda_g$ with respective constant multiplicities $m_1, \ldots, m_g$. Such hypersurfaces with $g \leq 3$ are classified due to Cartan's work [2] in 1939.

Note that the Bryant conjecture is very strong because M^3 is not assumed to be closed. Nevertheless, there is good evidence that it may be true. In [3], together with the works of Simons [11] and Peng-Terng [10], the author was able to establish the Chern Conjecture when $n=3$ by showing that each $M^3 \in \mathscr{M}^3_{\text{closed}}$ is an isoparametric hypersurface. Hence, $\mathscr{R}_3 = \{0, 3, 6\}$. Also, the Bryant Conjecture was verified