PACIFIC JOURNAL OF MATHEMATICS
Vol. 165, No. 1, 1994

THE STRUCTURE OF sl(2, 1)-SUPERSYMMETRY: IRREDUCIBLE REPRESENTATIONS AND PRIMITIVE IDEALS

DIDIER ARNAL, HEDI BEN AMOR AND GEORGES PINCZON

We give a detailed study of the enveloping algebra of the Lie superalgebra sl(2, 1), including classification of irreducible Harish-Chandra modules, completeness of finite dimensional irreducible, explicit computation of center, and classification of primitive ideals.

Introduction and main results. Lie superalgebras are important both in physics and in mathematics [**5**]. In physics, they are used e.g. to unify fermions and bosons in a unique picture (one irreducible representation of the structure) via supersymmetry. In mathematics, their enveloping algebras provide a class of very interesting noetherian algebras. Much information is known about enveloping algebras of Lie algebras (e.g., [**4**]), but for superalgebras there is a lot to do (see e.g. [**2**] for a pioneering work, and [**13**] for a very nice survey of results obtained up to now). Let us restrict to the simple case; then a natural distinction does appear between simple superalgebras with an enveloping algebra which is a domain and others. The first case is exactly the series $\mathrm{osp}(1, 2n)$, which are also the only semi-simple simple superalgebras [**8**]. The simplest model of this case is $h = \mathrm{osp}(1, 2)$; $U(h)$ was completely studied in [**16**], including explicit computation of Prim $U(h)$. The simplest model of the second case is $g = \mathrm{sl}(2, 1)$, and the purpose of the present paper is a complete study of $U(g)$. We shall give a classification of irreducible Harish-Chandra modules, a detailed computation of the center $Z(g)$ of $U(g)$, and a classification of $\mathrm{Prim}\, U(g)$.

Let us recall known results: finite dimensional irreducible representations of $g = \mathrm{sl}(2, 1)$ are known [**18**], and also unitary irreducible are classified ([**6**], [**7**]). Moreover, finite dimensional representations provide a complete set of representations [**2**], but are generally not fully reducible.

A fundamental result of our paper is the fact that finite dimensional irreducible provide a complete set, because of information that can be deduced on $U(g)$. Actually, we deduce an explicit determination of the center $Z(g)$, which shows that $Z(g)$ is not a finitely