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THE EFFECT OF DIMENSION ON CERTAIN GEOMETRIC PROBLEMS OF IRREGULARITIES OF DISTRIBUTION

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Suppose that equal numbers of red and blue points, all distinct, lie in the euclidean space $\mathbf{E}^t$, and consider a hyperplane h containing none of the points. If H is one of the open halfspaces determined by h, let $D(h)$ denote $|r(h) - b(h)|$ where $r(h)$ and $b(h)$ are the numbers of red and blue points lying in H. What can be said about the number $\sup D(h)$ as h ranges over all hyperplanes? The present article addresses this and similar problems of discrepancy principally by developing estimates of L^2 integral averages of $D(h)$ with respect to the invariant measure on the planesets of $\mathbf{E}^t$. Special attention is given to the influence of the dimension t.

The aim is to develop inequalities that involve only absolute constants and simple geometric properties of a given pointmass distribution. For example, the following theorem is an immediate corollary to more general results in this article.

THEOREM A. *Let $p_1, p_2, \ldots, p_N$ span $\mathbf{E}^t$ and be two-colored as described above. Then there is an absolute positive constant c such that*

$$\sup D(h) \geq c \max\{t, (\delta/\rho)^{1/2} t^{-1/4} [\min(\log N, t)]^{-3/4} \sqrt{N}\}$$

where δ is the minimum distance between distinct points and ρ is the maximum distance, or diameter, of the pointset.

The investigation continues that in **[A1]**, but it also draws upon a number of results in **[A2]**. The present work differs markedly from the earlier in that the dimension of the space is taken as a variable. This type of problem can be generalized to convex bodies other than halfspaces, but in this article we shall focus our attention on halfspaces. This seems to be a fundamental setting in which to study the relationship between irregularities of distribution and convexity. Moreover, the methods developed in **[A1]** and **[A2]** may be applied directly to this problem. For an excellent recent report on estimates of discrepancy concerning a wide variety of geometric shapes the reader is referred to the book **[BC]** by J. Beck and W. W. L. Chen.