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SOAP BUBBLES IN $\mathbb{R}^2$ AND IN SURFACES

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We prove existence and regularity for "soap bubbles" in $\mathbb{R}^2$ and in surfaces, i.e., the least-perimeter way to enclose and separate regions of prescribed area. They consist of constant-curvature arcs meeting in threes at 120 degrees. If one prescribes the combinatorial type too, then the arcs may bump up against each other.

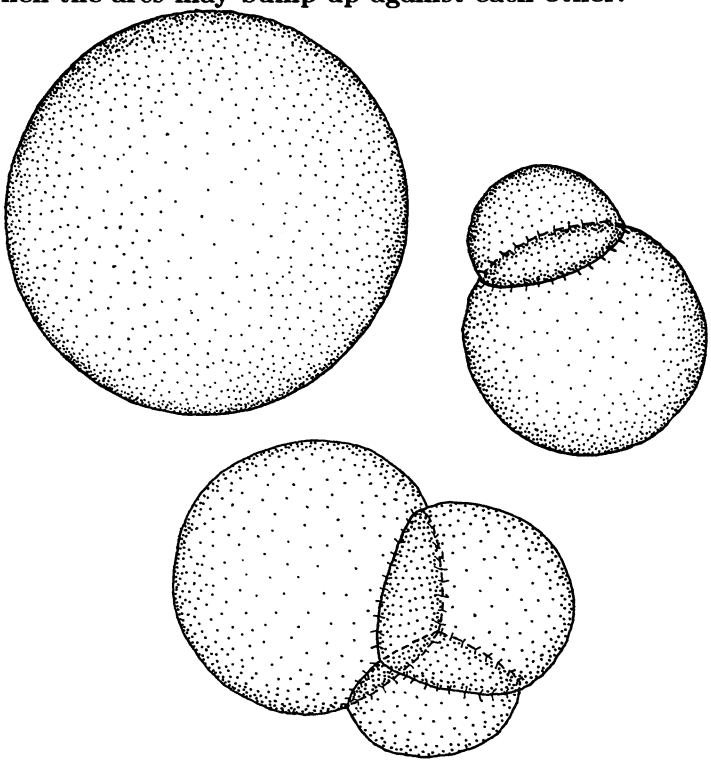

FIGURE 1.1. Single, double, and triple bubbles in $\mathbb{R}^3$ presumably provide the least-area way to enclose and separate the given volumes of air.
Drawings by J. Bredt [M5]