

ON THE DERIVED TOWERS OF CERTAIN
INCLUSIONS OF TYPE III_λ FACTORS OF INDEX 4

PHAN H. LOI

Given an inclusion of type III_λ factors, $\lambda \neq 0, 1$, of index 4 and with a common discrete decomposition, we compute the principal graph of its derived tower based on that on the associated type II_1 inclusion. Applications to the classification problem of hyperfinite type III_λ subfactors are discussed.

1. Introduction. Since the introduction and development of the theory of index by V. Jones in [J] to study a pair of type II_1 factors, one of the main problems has been the classification, up to conjugacy, of type II_1 subfactors of the same index of the hyperfinite type II_1 factor R_0 . Lately a great deal of progress has been made on this problem in [O1], [P1], [P2]. As these works show, the tower of higher relative commutants (also known as the derived tower) associated with an inclusion of type II_1 factors is an important conjugacy invariant finer than the index, and if the inclusion has finite depth, or more generally the generating property as introduced in [P2], then this invariant contains sufficient information to determine the subfactor completely.

In another development, the notion of an index has been extended by various authors, [Ko], [Ln], [PiPo1] to arbitrary inclusions of factors that are associated with a normal faithful conditional expectation.

In [L1], it is shown that the theory of index for type III_λ factors, $\lambda \neq 0, 1$, is closely related to that for type II_1 factors. In particular, when both factors are of type III_λ , $\lambda \neq 0, 1$, then such an inclusion can be studied by means of a common discrete decomposition. Motivated by the classification work on injective type III_λ factors