

VERMA MODULES INDUCED FROM NONSTANDARD BOREL
SUBALGEBRAS

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In this paper we study composition series and embeddings of Verma modules induced from “nonstandard” Borel subalgebras. This article can be viewed as a generalization of Futorny’s work on imaginary Verma modules for $A_1^{(1)}$ where the center of the Kac-Moody algebra acts nontrivially.

Introduction. Let A be an indecomposable symmetrizable generalized Cartan matrix, $\mathfrak{g}(A) = \mathfrak{n}_- \oplus \mathfrak{h} \oplus \mathfrak{n}_+$ the triangular decomposition of the Kac-Moody algebra $\mathfrak{g}(A)$ and W the Weyl group for $\mathfrak{g}(A)$. The “standard” Borel subalgebra \mathfrak{b}_+ and its opposite \mathfrak{b}_- are defined to be $\mathfrak{b}_\pm = \mathfrak{h} \oplus \mathfrak{n}_\pm$. For affine Kac-Moody algebras, H. Jakobsen and V. Kac and independently V. Futorny have found an explicit description of a set of representatives of the conjugacy classes of Borel subalgebras (see 1.1) under the action of $W \times \{\pm 1\}$. We will call all Borel subalgebras not conjugate to \mathfrak{b}_+ or \mathfrak{b}_- , “nonstandard” Borel subalgebras. In particular for each subset X of the set of simple roots Π for $\mathfrak{g}(A)$, one can construct a nonstandard Borel subalgebra \mathfrak{b}_+^X and then for each $\lambda \in \mathfrak{h}^*$ one can use induction to obtain what we will call a “nonstandard” Verma module $M^X(\lambda)$. For example if $X = \Pi$ then \mathfrak{b}_+^X is the standard Borel subalgebra and $M(\lambda)$ is the “standard” Verma module. At the other extreme $X = \emptyset$ one obtains the “natural” Borel subalgebra and what one might call a “natural” Verma module. A striking difference between Verma modules induced from a standard Borel and those induced from \mathfrak{b}_+^X for $X \subsetneq \Pi$, is that these new nonstandard Verma modules have infinite dimensional weight spaces. Consequently many of the classical techniques used in the study of the composition series of standard Verma modules do not seem to apply to this new setting.