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ENVELOPING ALGEBRAS AND REPRESENTATIONS OF TOROIDAL LIE ALGEBRAS

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This paper is about Toroidal Lie algebras which generalize the notion of an Affine Lie algebra. We study Verma type modules for these Toroidal algebras and prove an irreducibility criterion when the number of variables is two. We use the fact that the universal enveloping algebra is an Ore domain to obtain facts about the Verma type modules. Moreover, we are able to characterize the Affine Kac-Moody Lie algebras as those whose universal enveloping algebras are non-Noetherian Ore domains.

Introduction. A toroidal Lie algebra is a perfect central extension of the Lie algebra $\mathfrak{T}_{[m]}(\mathfrak{g}) = R_{[m]} \otimes \mathfrak{g}$ where $\mathfrak{g}$ is one of the finite dimensional simple Lie algebras over $\mathbb{C}$ and $R_{[m]}$ is the ring of Laurent polynomials in m variables $t_1, \ldots, t_m$ over $\mathbb{C}$. Here, the multiplication in $\mathfrak{T}_{[m]}(\mathfrak{g})$ is the obvious one defined componentwise. It turns out that these algebras are homomorphic images of some of the G.I.M. and I.M.Lie algebras defined by P. Slodowy (see [**2**], [**16**] and [**17**]) but it is not clear, at the outset, if there is a nontrivial kernel. In [**3**] realizations of certain of these I.M. Lie algebras are given (when $\mathfrak{g}$ is simply laced) and there it is shown, in a computational way using roots, that the kernel is non-trivial. A more conceptual way was sought by the present authors and we thought that, roughly speaking, the fact that the root spaces of $\mathfrak{T}_{[m]}(\mathfrak{g})$ have bounded dimension should be enough to allow one to see that the kernel is non-zero. This turns out to be true but much more is true as well.

Recall that in the paper [**15**] that fundamental use is made of the fact that for the affine algebras or the Virasoro algebras one has a root space decomposition with root spaces of bounded dimension,