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DEHN FILLING HYPERBOLIC 3-MANIFOLDS

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Define a complete family of parent (ancestor) manifolds to be a set of compact 3-manifolds such that every closed orientable 3-manifold can be obtained by one (or more) Dehn fillings of the manifolds in the family. In 1983, R. Myers proved that the set of 1-cusped hyperbolic 3-manifolds is a complete family of parent manifolds. We prove this result in a new way and then go on to prove:

THEOREM 1.1. *(a) Let V_0 be any positive real number. Then the set of 1-cusped hyperbolic 3-manifolds of volume greater than V_0 is a complete family of parent manifolds.*
(b) Let V_1 be any positive real number. Then the set of 1-cusped hyperbolic 3-manifolds of cusp volume greater than V_1 is a complete family of parent manifolds.
(c) The set of 2-cusped hyperbolic 3-manifolds containing embedded totally geodesic surfaces is a complete family of ancestor (actually grandparent) manifolds.
(d) For any positive integer N, the set of hyperbolic 3-manifolds, each of which shares its volume with N or more other hyperbolic 3-manifolds, is a complete family of ancestor manifolds.
As a corollary to Theorem 1(b), we prove that there exists a complete family of parent manifolds such that at most one Dehn filling on each manifold in the family yields a manifold of finite fundamental group.

1. Introduction. A *Dehn filling* of a 3-manifold with a torus boundary component is the procedure of gluing a solid torus to the 3-manifold along the torus boundary. If a manifold M is obtained from a manifold M' by a single Dehn filling, we will call M' a parent of M. If M is obtained from M' by some number of Dehn fillings, we will call M' an ancestor of M. We will define a set of 3-manifolds to