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CONJUGATES OF STRONGLY EQUIVARIANT MAPS

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Let $\tau: X_1 \to X_2$ be a strongly equivariant holomorphic embedding of one bounded symmetric domain into another. We show that if σ is an automorphism of C, then $\tau^\sigma : X_1^\sigma \to X_2^\sigma$ is also strongly equivariant.

1. Introduction. A semisimple algebraic group, G, over $\mathbf{Q}$, is said to be of hermitian type if $G(\mathbf{R})^0/K$ is a bounded symmetric domain for a maximal compact subgroup K of $G(\mathbf{R})^0$. Let X_1 and X_2 be bounded symmetric domains associated with algebraic groups G_1 and G_2 respectively. A holomorphic embedding $\tau: X_1 \to X_2$ is called weakly equivariant if there exists a homomorphism of algebraic groups $\rho: G_1 \to G_2$ defined over $\mathbf{Q}$, such that

(1.1) $\rho(g) \cdot \tau(x) = \tau(g \cdot x)$ for all $g \in G_1(\mathbf{R})^0$ and all $x \in X_1$.

τ is called strongly equivariant if, in addition, the image of X_1 is totally geodesic in X_2. It is not known, at least to me, whether every weakly equivariant holomorphic map of bounded symmetric domains is strongly equivariant.

Strongly equivariant maps form the central theme of Satake's book [**21**]. They have (at least) two important applications to number theory. If G_2 is a symplectic group, then X_2 parametrizes a universal family of abelian varieties. Pulling back the universal family to X_1, and taking the quotient by an arithmetic group, gives a family of abelian varieties called a Kuga fiber variety. These are defined in [**10**], where Kuga calls τ a (generalized) Eichler map. For the second application, to compactification of arithmetic varieties, see [**20**] and also [**4**], where strongly equivariant maps are called symmetric maps.