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ON A CONSTRUCTION OF PSEUDO-ANOSOV DIFFEOMORPHISMS BY SEQUENCES OF TRAIN TRACKS

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Pseudo-Anosov diffeomorphisms are representable by sequences of train tracks with some property. We introduce the necessary and sufficient condition for a sequence of train tracks under which the sequence represents a pseudo-Anosov diffeomorphism.

1. Introduction. Thurston compactified the Teichmüller space of a surface M by adding the measured foliation space $\mathcal{MF}$ as its boundary. Diffeomorphisms of M are classified by their natural actions on the compactified Teichmüller space. A diffeomorphism is called pseudo-Anosov when it acts so that an arational element of $\mathcal{MF}$, which has no connections of singularities, is invariant up to multiplication by a scalar not equal to 1 ([**1**]). Such a folation is called a pseudo-Anosov foliation.

A train track is a 1-dimensional CW complex drawn on M with a certain property (for example, see Fig. 8.4c). A train track is regarded as a coordinate system of $\mathcal{MF}$ by giving thickness to its edges (see Fig. 1.9). The coordinate transformations are piecewise linear. The set of all train track gives thus a PL structure on $\mathcal{MF}$ ([**3**]).

The operations split, shift and collapse give rise to a new train track from a train track (Fig. 1.6a). The coordinate neighborhood defined by one train track contains that of the other ([**6**], [**3**]). A sequence of so related train tracks, or, a sequence of such operations, is called a word. A pseudo-Anosov foliation is characterized as the intersection of the coordinate neighborhoods of the train tracks of iterations of a certain word ([**4**]). On the other hand, an arbitrary pseudo-Anosov diffeomorphism is representable by a